Numerical Simulation of Wave Overtopping and Erosion of Levee

X. Rao[1,2], L. Li[1], F. Amini[1,*]
[1]Department of Civil and Environmental Engineering, Jackson State University, Jackson, MS, USA
[2]on leave from College of Water Conservancy and Hydropower Engineering, Hohai University, Nanjing, China
*Corresponding Author

Abstract: In this paper, a purely Lagrangian and meshless approach, the smoothed particle hydrodynamics (SPH) method, was modified and combined with a physically-based erosion model adopted from an Eulerian approach to explore its ability to solve shallow water equation under combined wave overtopping and surge overflow conditions and incorporating corresponding hydraulic erosion of an earthen levee. Under combined wave and surge overtopping condition, the water velocity in the landward-side levee slope can cause severe soil erosion for the earthen levee. A method was developed to solve complicated landform problem in a levee, and a special technique was introduced to deal with the moving flow front. Ghost particles were assigned in the dealing with moving flow front boundary. Negative water depth and velocity of real particles were used for the ghost particles to keep the pressure and velocity of the boundary equal to zero. Linked-list algorithm was used as nearest neighboring particles searching algorithm. The eroded levee soil is associated with the SPH particles and is adverted due to the particle motion. The results show that the SPH method for shallow water flows and hydraulic erosion can provide practical solution to wave overtopping and erosion for a levee.

1. INTRODUCTION

The smoothed particle hydrodynamics (SPH) method was first introduced by Gingold and Monaghan (1977) to simulate astrophysical problems. As a Lagrangian and meshless method, SPH has been widely used in the field of free surface flow, such as solving shallow-water equations (Raidh 2005).

The SPH method is a meshless Lagrangian method that uses a pseudo-particle interpolation algorithm to calculate smooth field variables. Each pseudo- particle has a mass, Lagrangian position, Lagrangian velocity, and internal energy. Other quantities can be derived by interpolation or developed from constitutive relations. The pseudo-particles move with the velocity of the continuum, but are not associated with a grid and consequently do not have fixed connectivity (Liu and Liu 2003).

Levee breach problems are often numerically solved using Eulerian approaches such as the finite volume or finite element methods. Since the levee breach usually occurs under wave condition, and also as a result of the wetting – drying flood phenomena, the SPH method is more appropriate. In this paper, the physically-based erosion SPH model was introduced to couple erosion simulation with flow. Then the SPH method was used to simulate the levee breach process, under combined wave overtopping and surge overflow condition.

2. THEORY

2.1 Hydraulic Formulation

The inviscid SWE in the non-conservative form neglecting the bed slope and friction terms is written as:

[1] $$\frac{Dh}{Dt} + h\nabla\vec{u} = 0$$

[2] $$\frac{D\vec{u}}{Dt} + g\nabla h = 0$$

where h, u and g are respectively water height, depth-averaged velocity, and gravity. D/Dt refers to the total derivative.

The SPH discrete form of the momentum equations is obtained as (Raidh 2005):

[3] $$\frac{D\vec{u}_i}{Dt} = -\sum_{j=1}^{N} v_j (g(h_j + h_i) + \prod_{ij})\nabla w(x_i - x_j)$$

where h_i is the nodal water height of particle i.

In order to avoid the oscillations near shocks and interpenetration of particles, artificial viscosity was proposed by Monaghan (1986) and is given by

[4] $$\Pi_{ij} = \begin{cases} -\alpha_{ij}\tilde{c}_{ij}\mu_{ij} + \beta\mu_{ij}^2 & (u_i - u_j)(x_i - x_j) < 0 \\ 0 & \text{elsewhere} \end{cases}$$

where α and β are constants, $\tilde{c}_{ij}$ is the average of wave speed associated with particles i and j and $\mu_{ij} = (u_i - u_j)(x_i - x_j)$. The parameters α and β are often taken as 1.0 (Lattanzio et al. 1986, Monaghan 1986, Rao et al. 2009).

The continuity equation is implicitly satisfied since a Lagrangian kinematic approach is used, i.e. the particle masses are conserved. Water depth h can be computed using an SPH approximation as:

[5] $$h(x_i) = \sum_{j=1}^{N} v_j h_j w(x - x_j) = \sum_{j=1}^{N} m_j w(x - x_j)$$

Where m_j is the mass of particle j

2.2 Hydraulic Erosion

Shear stress is obtained as (Kristof et al. 2009):

[6] $$\tau = k\theta^n$$

Where τ is the shear stress, *k* is the shear stress constant, *θ* is the shear rate and *n* is the flow behaviour index.

The shear rate can be approximated as:

[7] $$\theta = v_{rel} / l$$

Where *l* is the distance between fluid and boundary.

The erosion rate is formulated as:

[8] $\varepsilon = k_{\varepsilon}(\tau - \tau_c)$

Where k_ε is the erosion strength and τ_c is the critical shear stress.

The SPH discrete form of mass change is expressed as (Kristof et al. 2009):

[9] $$\frac{dM_b}{dt} = -\sum_j L_b^2 \varepsilon(j)$$

2.3 Boundary Treatment Method

One of major boundary treatment methods in SPH is ghost (virtual) particle technique (Liu et al. 2002). The principle of dealing with boundary is by making the kernel to have a compact support, i.e., full of particles in the smoothing circle (Fig. 1a). Near the boundary, the kernel does not have complete smoothing circle. Symmetrical ghost particles needed to be added to make the smoothing circle completely full. Depending on the boundary conditions, ghost particles were assigned with different properties in velocity, mass, and volume. In this study, ghost particles were assigned in the dealing with moving flow front boundary. Negative water depth and velocity of real particles were used for the ghost particles to keep the pressure and velocity of the boundary equal to zero.

For the solid boundary, another type of virtual particles (type II) is set up on the boundary as the real particles approach the solid boundary (Fig. 1b). A force is produced strong enough to prevent interpenetration of particles through the solid boundary. The force is along the centerline between virtual and real particles, and points to the real particles.

2.4 Slope Landform

In the levee breach simulation, the landform of the levee during the breach changes over time. The bed slope is divided into many fixed particles. Each particle has its own properties: mass, height, volume, velocity, and acceleration. The height is the height of slope at each particle location. If the heights are different from others, the masses are different homologically. The velocity and acceleration of these fixed particles are equal to zero forever. The fixed particles were placed on the boundary of the bed slope (Fig. 2). In the erosion simulation, the mass of fixed particles will be calculated in each time step. When the mass decreases over time, the erosion occurs. The decreased mass is the eroded mass.

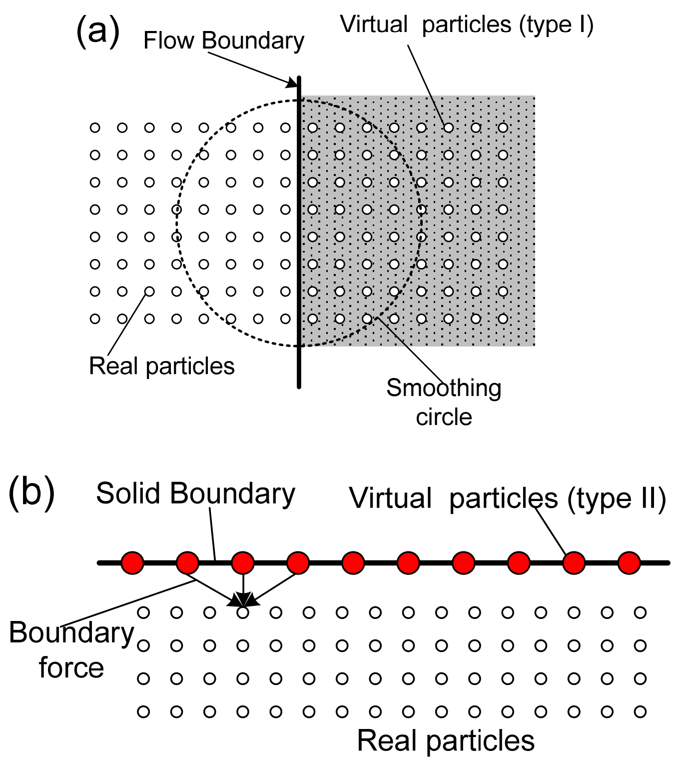


Figure 1. Boundary treatment method in this study: (a) ghost particles method, and (b) virtual particles in solid boundary

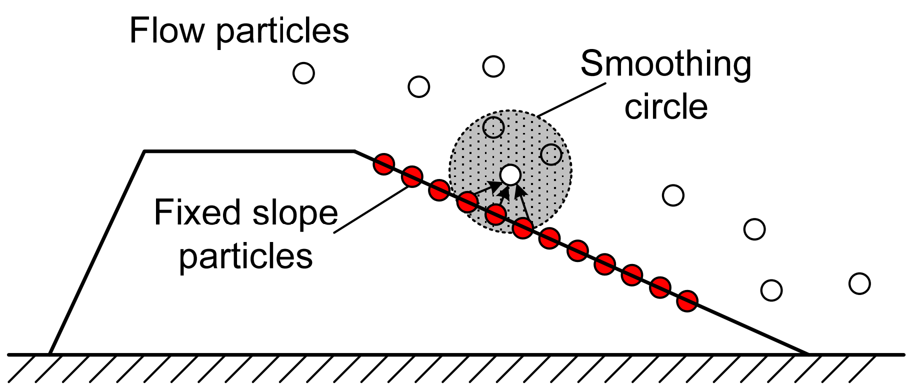


Figure.2. Setting up of slope particles

2.5 Nearest Neighboring Particles Searching Algorithm

In SPH, the distance between the particles varies in time. Thus, at each time step, the nearest neighboring particles for every specified particle have to be found. There are three searching methods: all-pair search algorithm, linked-list algorithm, and tree search algorithm (Liu et al. 2002). Fig. 3 illustrates the three nearest neighboring particles searching algorithm. Compared to all-pair search algorithm, linked-list algorithm is more effective and easy to implement. Tree search is more difficult to implement, but it has advantages for complex cases. In this study, the linked-list algorithm was used.

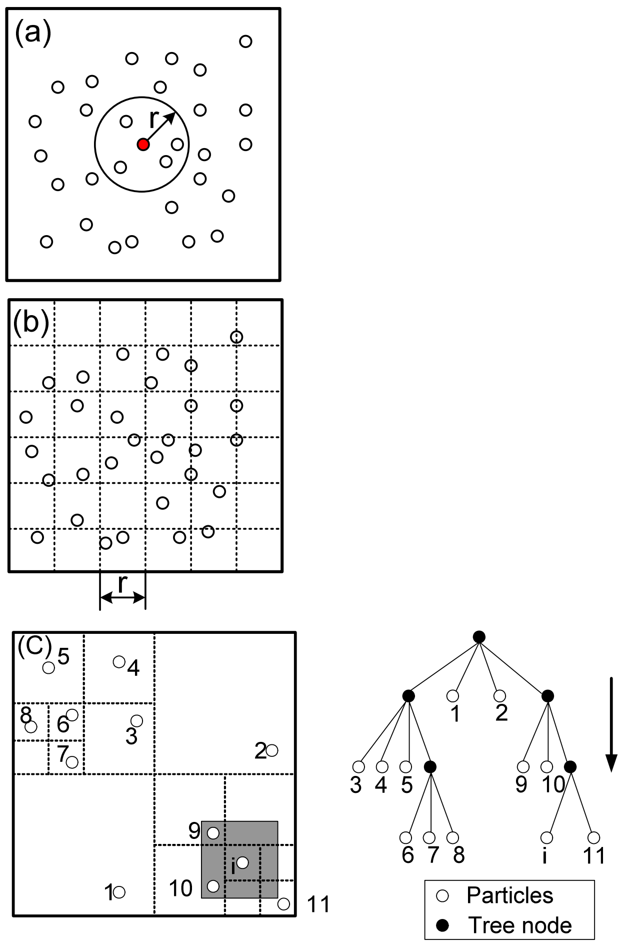
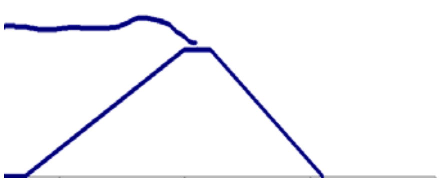


Figure 3. Algorithm of all-pair search (a), link-listed search (b), and tree search (c)

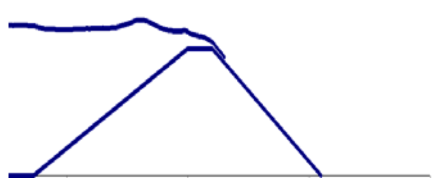
3. RESULT

In this study, a one-dimensional conceptual model at a levee was used to investigate the applicability of SPH method for a combined wave overtopping and surge overflow and hydraulic erosion process. The height of the levee is 3 m and the crest width is 2 m. The water-side slope is 4.25H:1V and the land-side slope is 3H:1V. An initial water level of 3.3 m is specified creating 0.3 m of surge overflow. A constant inflow discharge of 0.05 m^3/s is specified on the left boundary of the rectangular solution domain. The initial velocity was set as 0.1 m/s. A solitary wave with wave height of 0.3 m, wave width of 10 m, and peak wave velocity of 0.57 m/s was used to combine with the 0.3 m of surge overflow.

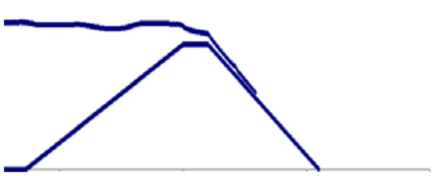
Fig. 4 shows the instantaneous free surface profiles at six different time instants under combined wave overtopping and surge overflow conditions. The hydraulic erosion is not considered in this example. At time T = 0.25 s, the water is confined to the left hand side of the levee but the water level is 0.3 m above the crest top and the solitary wave arrives on the crest. Due to the presence



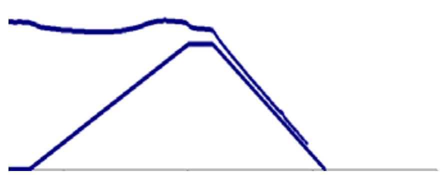
(a) T = 0.25 s



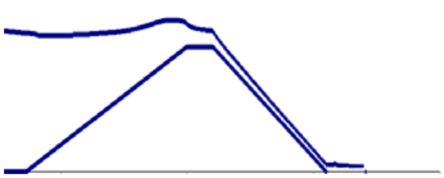
(b) T = 0.75 s



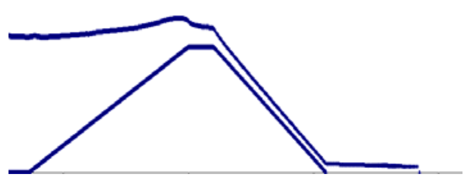
(c) T = 1.25 s



(d) T = 1.75 s



(e) T = 2.25 s

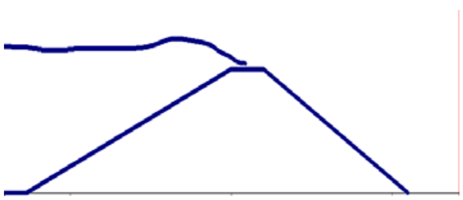


(f) T = 2.75 s

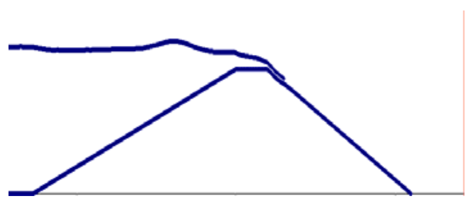
Figure 4. Combined surge overflow and wave overtopping at levee under various time: (a) T = 0.25 s; (b) T = 0.75 s; (c) T = 1.25 s; (d) T = 1.75 s; (e) T = 2.25 s; and (f) T = 2.75 s

of combined wave overtopping and surge overflow, the flow is pushed over the top of the levee crest, and the water level at the water-side slope of the levee is rising during the initial state of simulation. On the land-side slope of the levee, the water flow accelerates rapidly due to gravitational effect. The water depth near the toe of the levee is considerably shallower than that on the levee crest as the overtopping water flow velocity increases along the levee surface under the effect of gravity.

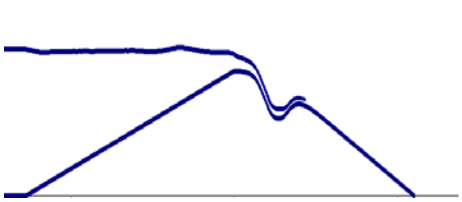
Fig. 5 shows the instantaneous free surface profiles and hydraulic eroded levee profile at six different time instants under the combined wave overtopping and surge overflow condition. In this case, the hydraulic erosion was included. To illustrate the hydraulic erosion process, relative high erosion parameters were used to avoid the concussion near the boundary. When the combined wave overtopping and surge overflow passed the crest and landed on the land-side slope, the accelerated water flow caused higher erosion stress on the landside slope. When the erosion stress exceeded the critical shear stress, erosion occurred on the slope, as shown on Fig. 5c.



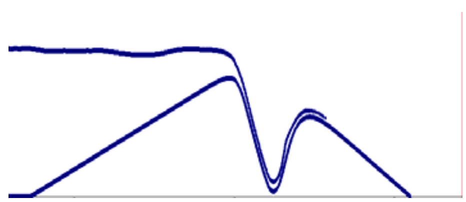
(a) T = 0.25 s



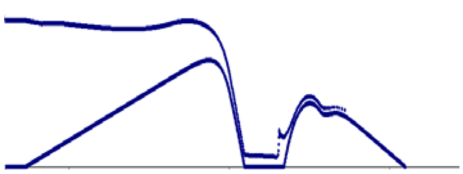
(b) T = 0.75 s



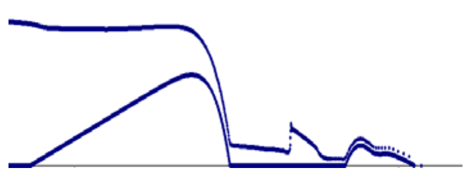
(c) T = 1.00 s



(d) T = 1.25 s



(e) T = 1.50 s



(f) T = 2.00 s

Figure 5. Levee breach under combined surge overflow and wave overtopping condition at various time: (a) T = 0.25 s; (b) T = 0.75 s; (c) T = 1.0 s; (d) T = 1.25 s; (e) T = 1.5 s; and (f) T = 2.0 s

The erosion extended from the landside slope to the top of the crest. When the top of the crest was eroded, the crest and land-side slope of the levee during the levee breech process were continuously eroded. After T = 1.5 s, the landside slope was eroded completely.

4. CONCLUSIONS

In this study, the SPH was modified to simulate combined wave overtopping and surge overflow for a levee. The hydraulic erosion caused by the combined wave overtopping and surge overflow was included. The combination of hydraulic erosion and overtopping for a levee was developed in the modified SPH model. New techniques were used to solve the complicated engineering flow. Results show that the modified SPH method can play an important role for simulating wave transmit problems and can solve combined wave transmit and erosion problems.

5. ACKNOWLEDGEMENT

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