A network flow based optimization approach for hurricane evacuation planning

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Abstract

We address the problem of finding the evacuation route, flow, and schedule in a short-notice evacuation problem due to natural disasters such as hurricanes. We formulate this problem as a network flow optimization model. Our goal is to find shortest paths in time and their flows to maximize the total number of evacuees from the danger areas to safe destinations. Since the corresponding optimization model is too large to solve on a PC, a heuristic algorithm has been developed to facilitate the solution process. This algorithm provides an efficient evacuation plan that includes optimum evacuation route, flow, and also schedule for every node in the evacuation area. An important characteristic of our model is that the flow rate of a path is assumed to be constant although each path is allowed to have a different flow rate. This is different than other approaches proposed in the literature. In fact, it is much easier for emergency management staff to implement a fixed rate flow plan than a variable evacuation flow rate in practice. We present some numerical experiments to show the strengths of our approach.

Keywords: Evacuation Planning, Network Flow Optimization, Shortest Path, Maximum Flow.

1. Introduction

2.1 Background

Hurricane is a rotating oceanic weather system with the wind speed exceeding 74 miles per hour. Impacts of hurricanes are severe and in many cases devastating. Recent hurricanes in the U.S. Gulf Coast areas such as Katrina and Rita truly evidenced that we are in dire need for a well-planned strategy in effectively evacuating large metropolitan areas. There were massive traffic congestions resulting from simultaneous evacuation of several million residents coupled with significant shortages of fuel and other basic necessities on the evacuation corridors. In the last three decades, hurricanes have increased in duration and intensity. With this trend, larger geographic areas and population segments will be affected by this weather phenomenon. Our preliminary analysis of the aftermath of hurricane Rita in Houston revealed two main shortcomings in Metropolitan evacuation planning procedure. First, there was *inadequate communication protocol to the public regarding evacuation plans.* The evacuees were neither adequately informed, nor clearly and specifically instructed about the evacuation procedure for both routing and scheduling. Evacuees did not know exactly which path to take for a safe and quick evacuation. The second was categorized as *absence of logistics support to evacuees.* The evacuation plan did not consider the needs of evacuees stranded in traffic congestion. No preparations to provide on-road access to basic amenities and sanitary services were made and vehicles lined up in front of gas stations as they ran out of fuel.

2.2 Literature Review

The main objective in an evacuation planning is to assign evacuation routes and schedules for evacuees so that a safe and timely evacuation plan can be seamlessly executed. Mathematical models for solving this type of problems are categorized as static network models, dynamic network models, traffic assignment models, and simulation models. Most of the evacuation models belong to dynamic network models. Ford and Fulkerson [3] developed *temporary repeated approach* that some instances of the maximum dynamic flow problem can be solved as a single static minimum cost problem. Wilkinson [8] modified the Ford-Fulkerson algorithm for maximal dynamic flow in a time weighed capacitated network. Kotnyek [5] presented an annotated overview of dynamic flows. Following this, Wilkinson [8] and Minieka [7] modified Ford and Fulkerson's repeated chain approach to obtain an earliest arrival flow. Hoppe and Tardos [4] presented a review for some polynomial time algorithms for evacuation problems.
Network Optimization or Network Flow problems have been also widely used in regional evacuation planning.

Ahuja *et al.* [1] summarized various applications of network flow problems. A classic application of network optimization is the problem of entity routing and scheduling. Yamada [10] used network flow concepts to model emergency city evacuations. Another category of solution approach for the evacuation problem is mixed integer programming (MIP). Cova and Johnson [2] proposed a model which is an extension to the minimum cost flow problem. Their model has two objectives. The first objective is to route vehicles to the nearest safe zones and the second one is to minimize the crossing conflict and intersection merging. Huang *et al.* [6] proposed a capacity constrained routing approach for evacuation planning. In this paper, capacity is modeled as a time series and a capacity constrained heuristic routing approach is used to solve the evacuation problem. Wilmot and Mei [9] conducted a study to compare the relative accuracy of alternate forms of trip generation for evacuation traffic. Yi-Chang Chiu *et al.* (2007) developed a no-notice mass evacuation model using dynamic traffic flow optimization. Note that a *no-notice mass evacuation* is defined as the evacuation that takes place immediately after the occurrence of a disaster event.

One of the main components addressed in the literature is to provide a variable evacuation flow rate over time for each evacuation route. However, it is a bit difficult to implement the solution of those approaches. In managerial point of view, a fixed (or constant) evacuation flow rate is much easier to implement. For example, most evacuation vehicles are subjected to follow the major highways. With a fixed flow rate, one can easily control how many vehicles can enter the high way via the ramp. It is simpler to implement. Therefore, we develop a mathematical framework for short notice mass evacuation using a fixed evacuation flow rate in this paper. Short-notice disasters are those that have a desirable lead time of between 24–72 hours allowing Emergency Management Agencies (EMAs) to determine alternate evacuation strategies *a priori* based upon the expected impacts of the disaster. Hence, our research attempt is to develop an optimization model that can provide efficient solutions in a timely manner to the short notice evacuation problems. We assume that decision makers have complete information on the number of evacuees in each area, the capacity and topology of transportation networks and the approaching hurricane's projected path. Based on the available information, we aim at facilitating the evacuation process by providing clear (easy to understand and easy to implement) temporal and spatial schedules and routes to evacuees by utilizing network optimization techniques.

The rest of the paper is organized as follows. A mathematical formulation and a solution algorithm are discussed in Section 2. In Section 3, the performance of the proposed algorithms on a few numerical case studies is presented. Finally, we summarize the paper and give possible future research venues in Section 4.

2. Methodology

2.1. Notation and assumptions

In this section, the mathematical notations used in our models are introduced (see Table 1), and a few assumptions made in this paper are discussed. We first consider our problem on a static network $G = (N, A)$ that represents the transportation network in the area of interest.

Notation	Description
N_d	set of all impact nodes
N_s	set of all safe nodes
$N = (N_d \cup N_s)$	set of all nodes
A	set of all arcs in the network
t_i	impact time at node i
s_i	initial number of evacuees located at node i
UN_i	maximum number of evacuees which can be located at node i per time period
tr_{ij}	travel time on the connecting road between node i and node j, ($tr_{ii} = 1$)
UA_{ij}	maximum number of evacuees which can enter into arc (i, j) per time period
Node $i \in N$	physical location including impact nodes and safe nodes
Arc $(i, j) \in A$	connecting road between node i and node

Table 1: Static network notation

Because of the nature of the evacuation problem, we divide the set of nodes N into two subsets: N_d (impact nodes) and N_s (safe nodes). The former includes all the nodes that are declared as evacuation zones due to the proximity of the projected hurricane path as well as the projected flood elevation level. The latter contains all the safe nodes at which people are trying to reach while evacuating. Nodes are further classified as *centroids* or *intersections*. Centroids are the nodes with positive supplies whereas intersections do not hold supplies. Furthermore, we add one parameter to the nodes of the network, t_i, which is the impact time of node i. It is defined as the amount of time that is available for people in node i to evacuate until the impact of the projected hurricane is made to the area. By definition, the impact time for a safe node is equal to infinity because safe nodes are not supposed to be affected by hurricane. Other parameters are self-explanatory and definitions are given in Table 1.

Evacuation networks are typically large. But if we add the time component to the optimization problem, the corresponding model becomes extremely large scale that there are no known polynomial algorithms for solving such problems. Problems can get far more complicated if the model considers uncertain factors that can affect the performance of evacuation. Such factors may include uncertain human behavior to evacuation, availability of resources for evacuation, uncertain travel times, uncertain behavior of hurricane trajectory and intensity, to name a few. Therefore, the following assumptions are made for tractability of the considered problem in this paper.

1. *Human evacuation behavior and evacuee's limitations for evacuation are not considered:* In reality, no one can predict human behavior to the evacuation. Some people may have decided not to evacuate or some evacuees may not follow the evacuation instructions given by the local evacuation management officials. Such a psychological aspect of evacuation is hard to capture in the model. Therefore, it is not considered in this paper.
2. *Constant hurricane propagation speed and known hurricane path:* The dynamic nature of the hurricane propagation is not taken into account. Thus, we assume that the hurricane follows the projected hurricane path obtained at least three days prior to the landfall and the speed of hurricane is assumed to be constant during this time.
3. *Constant transit times:* The transit time on each arc of the network is assumed constant and known *a priori*. In other words, the phenomenon of congestion on the road network is not considered in this paper.
4. *Evacuation region definition:* The impact nodes in the network are categorized into different regions based on their distance from the flood elevation level so that people who live closer to a high flooding area receive a higher priority for evacuation. The number of evacuation regions depends on the size of the network and the number of nodes in the network.

2.2. Dynamic Network Construction

Because the nature of any evacuation problem is dynamic, we first construct a dynamic network (or time expanded network) that expands the static network over the planning horizon for every time interval. Note that both the terms are used interchageably in this paper. Time-expanded networks were introduced by Ford and Fulkerson [3] in the fifties. They introduced a computational algorithm to maximize the flow from the source node to the sink node in a network within a given time period T. We adopt their method to construct our time-expanded network, *i.e.*, all the nodes and the arcs of the static network are duplicated at each time period. The resulting arcs are called movement arcs. There are also holdover arcs for each node which hold the flow of that node for one time period.

We slightly modify the mathematical notation for constructing the time-expanded network. Let $G^T = (N^T, A^T)$ represent a time-expanded network of a static network $G = (N,A)$ over a T evacuation planning horizon. Where, $N^T = \{i_t | i \in N; t = 1, \ldots, T\}$ is the set of nodes in the time-expanded network, $A_M = \{(i_t, j_{t+trij}) | (i,j) \in A; t + tr_{ij} \leq T; t = 1, \ldots, T\}$ is the set of all movement arcs while $A_H = \{(i_t, i_{t+1}) | i \in N; t = 1, \ldots, T-1\}$ is the set of all holdover arcs. Therefore, the set of arcs in the time-expanded network is the union of the two arc sets, *i.e.*, $A^T = \{A_M \cup A_H\}$. In addition, the capacity of an arc (a, b), $U_{a,b}$, in the time-expanded network is

$$U_{a,b} = \begin{cases} UA_{ij}\ , \ if\ \ (a,\ b) \in A_M \text{ and } a = i_t,\ b = j_{t+tr_{ij}} \text{ for some } t \in \{1, \ldots, T\} \\ UN_i,\ if\ \ (a,\ b) \in A_H \text{ and } a = i_t,\ b = t_{t+1} \text{ for some } t \in \{1, \ldots, T-1\} \end{cases}$$

Note that the flow of an arc (i, j) is bounded by its own arc capacity UA_{ij} if it is a movement arc at node i at time t. If the arc is an holdover arc, it is just an imaginary arc that represents the supply of the node from time t to $t + 1$. Therefore, the flow of such an arc (i_t, i_{t+1}) is bounded by its capacity (supply at node i) at time t, UN_i. Furthermore, two imaginary nodes J^* ('Super-safe' node), and J' ('Unsafe' node) are added to G^T. Then, each safe node i_t, $\forall i \in$

N_s, for each $t = 1, ..., T$ in G^T is connected to node J* through the arc (i_t, J*), $\forall i \in N_s$, with capacity $U_{it,J^*}=UN_{it}$, and each node i_1, $\forall i \in N_d$, in G^T is connected to node J′ through arc (i_1, J′), $\forall i \in N_d$, with capacity $U_{i1,J'}=b_{i1}$. The Super-safe node is assumed to be the final destination for all the evacuees, while Unsafe node is considered as a shelter location for people who could not evacuate.
Because of the enormous size of the time-expanded network especially for large scale problems, we attempt to reduce the size of the time-expanded network. Since people have t_i time to evacuate, node i and all its connecting arcs should not be in the network after time t_i. So, we need to delete all arcs (i_t, j_{t+trij}), $\forall$ t, for which $t + tr_{ij} \geq t_j$ from G^T, and also all arcs (i_t, j_{t+trij}), $\forall t$, such that $t \geq t_i$. Since node i will be impacted at time t_i, we delete all nodes i_t, $\forall i \in N_d$, $t \geq t_i$ from G^T. Furthermore, since the intersection nodes do not hold any population, we delete all the holdover arcs that are associated with an intersection node.

2.3 Multicommodity Network Flow Problem (MCNFP) Formulation

The main optimization model in this paper is formulated as a multicommodity network flow problem. MCNFP is a problem of sending flows of each commodity from its origin to the destination while sharing the capacity of the arcs with other commodities. The MCNFP model is designed to determine the optimal evacuation route and schedule for each commodity in the network. For our problem, a commodity is defined as an evacuation path of a node. Note that each node is allowed to have multiple paths (or commodities). An extensive survey of the multi-commodity flow problems can be found in [1]. In this paper, we present a maximum flow multi-commodity network flow problem which satisfies balance constraints and capacity constraints. Suppose that we have R regions in the network each of which consists of a subset of impact nodes. Each impact node in region $r \in \{1,\cdots,R\}$ has one or more commodities going out of it. The methodology to define the commodity weights are fully described in [7].

Given such commodity definition, our decision variables to the optimization model are:

x_{itjk} = The number of evacuees leaving node i to node j at time t using commodity k.

$$y_{ijk} = \begin{cases} 1, \text{if arc } (i, j) \text{ belongs to the path of commodity } k, \\ 0, \text{otherwise.} \end{cases}$$

b_{ik} = The number of people leaving node i choosing the path of commodity k.

Our objective is to maximize the total number of evacuees (or flow) for all commodities from the R regions to safe destinations. Depending on the urgency of the evacuation, each region is assigned with a different weight. If a region r needs to be evacuated first, the highest weight will be assigned to the region, and *vice versa*. Considering $\delta(i)$ as the set of commodities originating from node i, the following is the optimization model for our multi-commodity network flow problem:

$$Maximize \quad z_{mcnfp} = \sum_{i \in N_s} \sum_{k} \sum_{t} x_{itj^*k} \times \hat{w}_k^{\,r} \qquad (2.1)$$

Subject to

$$\sum_{k \in \delta(i)} b_{ik} = s_i \quad \forall i \in N \qquad (2.2)$$

$$\sum_{j | j=J' \ or \ (i_t, j_{t+tr_{ij}}) \in A_T} x_{itjk} = b_{ik}, \qquad \forall i \in N_d,\ t=1,\ \forall k \qquad (2.3)$$

$$\sum_{j | (i_t, j_{t+tr_{ij}}) \in A_T} x_{itjk} = b_{ik}, \qquad \forall i \in N_s,\ t=1,\ \forall k \qquad (2.4)$$

$$\sum_{j | j=J^* or\, (i_t, j_{t+tr_{ij}}) \in A_T} x_{itjk} - \sum_{j | (j_{t-tr_{ji}}, i_t) \in A_T} x_{j(t-tr_{ji})ik} = 0, \qquad \forall i \in N,\ t=2,...,T-1,\ \forall k \qquad (2.5)$$

$$\sum_{k} x_{itjk} \leq UA_{ij} \qquad \forall (i,j) \in A,\ t=1,...,T \qquad (2.6)$$

$$\sum_{k} x_{itik} \leq UN_{ij} \qquad \forall i \in N,\ t=1,...,T \qquad (2.7)$$

$$\sum_{j|(i,j)\in A} y_{ijk} - \sum_{j|(j,i)\in A} y_{jik} = 1, \quad \forall i \in N_d,\ \forall k,\ \ if\ \ i = \text{source}(k) \tag{2.8}$$

$$\sum_{j|(i,j)\in A} y_{ijk} - \sum_{j|(j,i)\in A} y_{jik} = 0, \quad \forall i \in N,\ \forall k,\ \ if\ \ i \neq \text{source}(k) \text{ or } \text{sink}(k) \tag{2.9}$$

$$\sum_{j|(i,j)\in A} y_{ijk} - \sum_{j|(j,i)\in A} y_{jik} = -1, \quad \forall i \in N_s,\ \forall k,\ \ if\ \ i = \text{sink}(k) \tag{2.10}$$

$$x_{itjk} \leq UA_{ij} \times y_{ijk}, \qquad \forall (i,j) \in A,\ t = 1,...,T\ ,\forall k \tag{2.11}$$

$$x_{itik} = 0, \qquad if\ i \neq \text{source}(k),\ t = 1,...,T\ ,\forall k \tag{2.12}$$

$$x_{itjk} \geq 0,\ y_{ijk} \in (0,1),\ \forall (i,j) \in A,\ t = 1,\ldots,T,\ \forall k. \tag{2.13}$$

$$x_{itik} \geq 0,\ \forall i \in N,\ t = 1,\ldots,T,\ \forall k. \tag{2.14}$$

$$x_{i1J^*k} \geq 0,\ \forall i \in N_d,\ \forall k. \tag{2.15}$$

$$x_{itJ^*k} \geq 0,\ \forall i \in N_s,\ t = 1,\ldots,T,\ \forall k. \tag{2.16}$$

3. Solution Method

To overcome the computational complexity of solving the MCNFP model discussed in Section 3, we propose a heuristic algorithm that provides an evacuation path for each node assuming a constant evacuation flow rate over a planning horizon and a non-interrupted evacuation schedule. The objective of our heuristic is to find the shortest evacuation path from each impact node to the safe area, and send the maximum possible flow through this path. A dummy safe node < d > is added to the static network with an infinite capacity, an infinite danger time, and a zero supply. Then, all the safe nodes are connected to this dummy safe node through arcs with infinite capacity and zero traveling time. These dummy nodes and arcs are used to find the shortest distance from impact nodes to the closest safe node using the *Dijkstra's algorithm*. Given the shortest path, the *Ford-Fulkerson algorithm* is applied on time-expanded network of this shortest path to obtain the maximum possible number of evacuees that can flow through this shortest path. If this path does not have enough capacity to evacuate all the evacuees coming out from that impact node, a second shortest path will be generated, and so on. This heuristic algorithm is easier to implement than the solution method we presented in [6]. This new approach considers a single evacuation route for each impact node and a fixed flow rate for each route. Also, the flow of each path remains constant at all time intervals, and the evacuation schedule is designed that once the evacuation starts at a specific time from one node, and it continues until all evacuees are transported to safe areas. Note that the MCNFP model does not satisfy these conditions.

4. Numerical result

We test our algorithm on nine case studies. These are randomly generated networks with different configurations. The proposed heuristic algorithm is implemented in C++ while the MCNFP model is solved using CPLEX 11.2. All experiments are made on a PC with 2.83GHz Intel Quad Xeon CPU and 16GB RAM running Windows Server 2008.

Table 2: Numerical Result

ID	Number of Nodes	Total number of cars	MCNFP		Heuristic	
			Evacuation %	Time (sec)	Evacuation %	Time (sec)
1	3	120	100%	0.234	100%	0.000
2	8	169	100%	0.251	100%	0.003
3	10	222	100%	0.360	100%	0.004
4	27	600	100%	86.613	100%	0.163
5	31	600	100%	245.453	100%	0.258
6	37	600	100%	73.875	100%	0.384
7	30	600	100%	129.589	100%	0.242
8	34	600	100%	41.530	100%	0.295
9	42	600	NA	NA	100%	0.539

The results are tabulated in Table 2. The first three experiments are categorized as small networks, while the rest of them are considered medium networks with the same amount of supplies, but a different number of nodes. Both the optimization model and the heuristic algorithm achieved 100% evacuation for the first eight test cases. Overall, the heuristic algorithm runs substantially faster than CPLEX for all cases tested. In some cases, our heuristic gets up to a 99% speed gain. In the case of ID=9, CPLEX failed to converge to a solution after 12 hours. Note that the heuristic algorithm finds the solutions to all cases in less than a second.

5. Conclusion

We have developed a multi-commodity network flow optimization problem for finding the evacuation route, flow, and schedule in a short-notice evacuation problem due to natural disasters. Due to the fact that evacuation takes place over time, we presented a time-expanded network approach to capture the dynamics of evacuation at different times. However, time-expanded networks grow exponentially in size as the number of nodes increases in the original network, for which there are no known polynomial algorithm for finding optimal solutions. Therefore, we have developed a novel heuristic algorithm that finds solutions within a small fraction of time that of the CPLEX computation time. A unique feature of our optimization model is that the flow rate of a path is assumed to be constant although each path is allowed to have a different flow rate. This scheme is simple and practical, yet, it has not been addressed in the literature. Overall, our model is easy to implement in practice and the solution algorithm is fast and robust for finding the evacuation plans.

Acknowledgement

This work has been supported part by the U.S. Department of Homeland Security.

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