A simple binary search algorithm for short notice evacuation scheduling and routing

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Abstract

Hurricanes are one of the costliest and most destructive events in the United States. Hundreds of people were either died or stranded in the impacted areas due to hurricanes recent years. Therefore, proper preparations can substantially reduce damages incurred by hurricanes. Evacuation is one of the crucial components in disaster preparation. It is essential to determine the precise routes and schedules in evacuation process. Executing the evacuation process too early may result in major economic losses in the evacuated areas and hamper delivery of critical supplies to the region under threat. Executing the evacuation process too late may result in tremendous traffic congestion and may subject many people to critical risks. In this paper, we present the mathematical models and a heuristic algorithm that can efficiently determine the starting times, schedules, and recommended routes of the evacuation process based on the estimated hurricane path and time of landfall information. The proposed methodologies have been applied to a number of evacuation problems. Our results show the effectiveness of the solutions generated by the proposed methods.

Keywords

Emergency Evacuation, Network Optimization, Max Flow.

1. Introduction

1.1 Background

Recent hurricanes have proven that the impact of natural disasters can be severe and in many cases devastating. The arrival of hurricanes Gustav, Katrina and Rita, in particular, has given real situations to local and federal agencies for testing their ability in evacuating and safely relocating their residents. The massive traffic congestion resulting from simultaneous evacuation of several million residents coupled with significant shortages of fuel and other basic necessities, and on the other hand assigning hundreds of buses for public transportation which were barely used by people, once again underscored the importance of a well-planned strategy in effectively evacuating large metropolitan areas. In the last three decades, hurricanes have increased in duration and intensity. If this trend continues, larger geographic areas and population segments will be affected with this weather phenomenon.

Our preliminary analysis of the aftermath of hurricane Gustav in Houston revealed two main shortcomings in Metropolitan evacuation planning procedure. First, there was "*inadequate communication protocol to public regarding evacuation plans.*" The evacuees were neither adequately informed, nor clearly and specifically instructed about the evacuation procedure for both routing and scheduling. People did not exactly know which path to take for a safe and quick evacuation. The second was categorized as "*absence of logistics support to evacuee.*" The evacuation plan did not consider the needs of evacuees stranded in traffic congestion. No preparations to provide on-road access to basic amenities and sanitary services were made and vehicles lined up in front of gas stations as they ran out of fuel.

1.2 Objective

In this paper, we focus our attention only on an evacuation problem that deals with assigning evacuation routes and schedules to evacuees in different living areas. In particular, we plan to address the strategic routing and scheduling problems faced by federal and local government that manage the evacuation operations of a large metropolitan area. We assume that decision makers have complete information on the number of evacuees in each area, the capacity and topology of transportation networks, and the path forecasting of approaching hurricanes. Such an assumption is for evacuation planning process in practice. Based on the available information, we aim at facilitating the evacuation process by providing clear (easy to understand and easy to implement) temporal and spatial schedules and routes to evacuees by utilizing network optimization techniques.

This paper is structured as follow. Section 2 reviews literature on evacuation planning. Mathematical formulations and proposed heuristic algorithm are fully discussed in Section 3. In Section 4, the performances of the proposed algorithms on a number of numerical case studies are summarized. Finally, we report the possible future researches and the summary of the paper in Section 5.

2. Literature Review

There are two types of evacuation problems; building evacuation and regional evacuation. We primarily focus on solving regional evacuation problems. Generally, the methods to model these problems are categorized as static network models, dynamic network models, traffic assignment models, and simulation models. Most of the evacuation models are dynamic network models.

Ford and Fulkerson [3] showed by an approach called temporary repeated approach that some instances of maximum dynamic flow problem can be solved through solving single static minimum cost problem. Wilkinson [8] modified the Ford-Fulkerson algorithm for maximal dynamic flow in a time weighed, capacitated network. Kotnyek [5] presented an annotated overview of dynamic flows. Following this, Wilkinson [8] and Minieka [7] modify Ford and Fulkerson's repeated chain approach to obtain an earliest arrival flow. Hoppe and Tardos [4] present a review for some polynomial time algorithms for evacuation problems.

There are so many approaches to solve the evacuation problems. For example Network Optimization or Network Flow problems are widely used in regional evacuation planning. Ahuja et al. [1] summarize the various applications of network flow problems. A classic application of network optimization is the problem of entity routing and scheduling. Yamada [10] used network flow concepts to model emergency city evacuations.

Another approach to solve evacuation problems is a mixed integer programming approach. Cova and Johnson [2] proposed a model which is an extension of the minimum cost flow problem. Their model has two objectives. The first objective is to route vehicles to the nearest safe zone and the second one is to minimize the crossing conflict and intersection merging. Huang et al. [6] proposed a capacity constrained routing approach for evacuation planning. In this paper, capacity is modeled as a time series and use a capacity constrained heuristic routing approach to solve the evacuation problem. Wilmot and Mei [9] conducted a study to compare the relative accuracy of alternate forms of trip generation for evacuation traffic. Conventional participation rate, logistic regression and various forms of neural networks were estimated and tested. Yi-Chang Chiu et al. (2007) developed a no-notice mass evacuation model using dynamic traffic flow optimization. A no-notice mass evacuation is defined as the evacuation that takes place immediately after the occurrence of a disaster event is defined as a "no-notice evacuation".

Our research attempt is to develop a mathematical framework for short notice mass evacuation as opposed to no-notice mass evacuation. Short-notice disasters are those that have a desirable lead time of between 24–72 hours allowing Emergency Management Agencies (EMAs) to determine alternate evacuation strategies a priori based upon the expected impacts of the disaster. Therefor, our research attempt aims at developing a model that provides quick solutions to the short notice evacuation problem using heuristic techniques.

3. Methodology

We start by introducing notations and assumptions that will be used in this paper. We consider a static network $G = (N,A)$ that represents the transportation network in the area of interest, and notations are summarized in Table 1.

Notation	Description
N_d	set of all possible impact zones
N_s	set of all possible safety zones
$N = (N_d \cup N_s)$	set of all zones
A	set of all arcs in the network
t_i	impact time at location i
b_i	initial number of evacuees located at location i
UN_i	maximum number of evacuees which can be located at location i per time period
tr_{ij}	travel time on the connecting road between location i and location j, ($tr_{ii} = 1$)
UA_{ij}	maximum number of evacuees which can enter into arc (i, j) per time period
Nodes $i, j \in N$	physical locations including impact zones and safety zones
Arc $(i, j) \in A$	connecting road between location i and location j

Table 1: static network notations

Let $G_T = (N_T,A_T)$ represent a time expanded network of a static network $G = (N,A)$ over T planning horizons with the notations shown on Table 2.

In this paper, the following assumptions are made for tractability of the considered problem.

i. *The behavior and the moving ability of evacuees are not considered.* This classification and human behavior are not taken into consideration in any of our optimization models.

ii. *Hurricane Propagation Speed and Characteristics:* The dynamic nature of the hurricane propagation is not taken into account. Thus, the planning time and the direction of the hurricane are assumed constant.

iii. *Transit times are assumed to be constant*

Based on these assumptions, our solution approach consists of four modules for making evacuation decisions. The first module is a linear programming (LP) model which determines the upper bound on the number of evacuees (P-Max) that can be safely evacuated from all impact zones within a given time horizon. In reality, the actual number of safely evacuated people will be less than or equal to this generated number due to traffic congestion and other unknown factors. Knowledge of this upper bound can help authorities in preliminary investigation of the feasibility of evacuation procedures and in planning for shelter locations and their capacities. The second module involves solving a number of LP models by utilizing the binary search algorithm. The result of this module is the lower bound on the total evacuation time. Knowledge of this lower bound again gives an insight to authorities in preliminary investigation of the feasibility of evacuation procedures and in planning shelter locations and capacities. The third and forth modules aim at finding simple and efficient evacuation routes and schedules. The former utilizes the mixed integer linear programming (MILP) model and the later utilizes the heuristic algorithm based on the network flow algorithms.

In the following subsections, we discuss each of these four modules in more detail.

Module 1: Calculating the Value of P-Max

This module consists of the following steps.

Step 1: *Constructing Time Expanded Network:* Given a static network $G = (N, A)$ and the planning horizon T, construct the time expanded network $G_T = (N_T, A_T)$. Add two imaginary nodes J^* called the 'Super Safe' node and J' called the 'Unsafe' node, to G_T.

Step 2: *Adding Dummy Nodes and Arcs:* Connect each safe node i_T $\forall i \in N_s$ in G_T to node J^* through arc (i_T, J^*) $\forall i \in N_s$ with capacity $Cap_{i_T,J^*} = UN_{i_T}$. Connect each node i_1 $\forall i \in N_d$ in G_T to node J' through arc (i_1, J') $\forall i \in N_d$ with capacity $Cap_{i_1,J'} = b_{i_1}$.

Step 3: *Preprocessing:* For each $t = 1, ..., T$, delete all arcs $(i_t, j_{t+tr_{ij}})$ for which $t + tr_{ij} \geq t_j$ from G_T. Delete all arcs $(i_t, j_{t+tr_{ij}})$ such that $t \geq t_i$ and delete all nodes i_t $\forall i \in N_d, t \geq t_i$ from G_T.

Notation	Description
T	available evacuation time horizon
$N_T = \{i_t \| i \in N; t = 1, \ldots, T\}$	set of nodes in the time expanded network
$A_M = \{(i_t, j_{\bar{t}}) \| (i,j) \in A; \bar{t} = t + tr_{ij} \leq T; t = 1, \ldots, T\}$	set of movement arcs
$A_H = \{(i_t, i_{t+i}) \| i \in N; t = 1, \ldots, T-1\}$	set of all holdover arcs
$A_T = (A_M \cup A_H)$	set of all arcs in the time expanded network
$Cap_{a,b} = UA_{ij}$ if $(a,b) \in A_M$ and $a = i_t$, $b = j_{\bar{t}}$ for some t and $\bar{t}$ in $\{1, \ldots, T\}$	
$Cap_{a,b} = UN_i$ if $(a,b) \in A_H$ and $a = i_t$, $b = i_{t+1}$ for some t in $\{1, \ldots, T-1\}$	

Table 2: static network notations

Step 4: Generate and solve the following LP model to obtain P-Max.

$$\text{Maximize} \quad \text{P-Max} = \sum_{i \in N_s} x_{iTJ^*} \tag{1}$$

Subject to:

$$\sum_{j|j=J' \text{ or } (i_t, j_{t+tr_{ij}}) \in A_T} x_{itj} = b_i \qquad \forall i \in N_d, t = 1 \tag{2}$$

$$\sum_{j|(i_t, j_{t+tr_{ij}}) \in A_T} x_{itj} = b_i \qquad \forall i \in N_s, t = 1 \tag{3}$$

$$\sum_{j|(i_t, j_{t+tr_{ij}}) \in A_T} x_{itj} - \sum_{j|(j_{t-tr_{ji}}, i_t) \in A_T} x_{j(t-tr_{ji})i} = 0 \qquad \forall i \in N, t \in \{2, \ldots, T-1\} \tag{4}$$

$$x_{itJ^*} - \sum_{j|(j_{t-tr_{ji}}, i_t) \in A_T} x_{j(t-tr_{ji})i} = 0 \qquad \forall i \in N_s, t = T \tag{5}$$

$$0 \leq x_{itj} \leq UA_{ij} \qquad \forall (i,j) \in A, t \in \{1, \ldots, T\} \tag{6}$$

$$0 \leq x_{iti} \leq UN_i \qquad \forall i \in N, t \in \{1, \ldots, T\} \tag{7}$$

$$x_{i1J'} \geq 0 \qquad \forall i \in N_d \tag{8}$$

$$x_{iTJ^*} \geq 0 \qquad \forall i \in N_s \tag{9}$$

where, x_{itj} is the number of evacuees leaving node i to node j at time t.

Module 2: Establishing the Lower Bound on Total Evacuation Time

The binary search algorithm is utilized in order to find the lower bound value on the total evacuation time for evacuating the P-Max evacuees to safety. In this module, we first construct a linear programming model by adding the following constraint

$$\sum_{i \in N_s} x_{iTJ^*} = \text{P-Max}$$

to the LP model discussed in Module 1. This LP model is solved iteratively for each iteration of the binary search algorithm with different setting of T. The value of time horizon T is varied over a range of values and run the optimization model for each value of T. The feasibility on each of the runs is checked. The minimum value of T for which the model proves to be feasible is the lower bound. In the binary search algorithm, two terms Upper Bound (UB) and Lower Bound (LB) are defined to establish a range of values over which the value of T is checked. We define a constant ε as the smallest possible integer difference between upper bound and lower bound.

Initially, we set the value of UB to T' where T' is the initial value of T and set the value of LB to zero. The optimization model is solved by setting T at UB. If the model is feasible, the algorithm reduces the value of UB to the current value of T. Otherwise, the algorithm reduces the value of LB to the current value of T. The algorithm then checks the difference between the values of UB and LB. If this value is greater than ε, the T value is set to $(UB + LB)/2$ and the same procedure is repeated. Otherwise, the algorithm is terminated and the current value of T is the lower bound value on the total evacuation time. Having knowledge of this lower bound helps authorities decide on evacuation methods. A small difference between this lower bound and the initial value of T indicates that the buffer time for unexpected events in the evacuation process is small. As a result, the authorities may have to plan alternate methods of evacuation in order to ensure safety for evacuees. On the other hand, if this difference is large, the authorities can plan the evacuation procedures accordingly. The evacuation can be planned so as to avoid confusion and panic among people.

Module 3: Developing the multi-commodity MILP Model

Since the output of the first module is confusing and difficult to follow and we cannot simply interpret the evacuation path from the output, we construct a multi-commodity MILP model in which the commodities are so useful for tracking the evacuation paths. Let us introduce decision variables

x_{itjk} = The number of evacuees leaving node i to node j at time t using commodity k
y_{ijk} = 1 if arc (i,j) belongs to the path of commodity k, zero otherwise
b_{ik} = The number of people leaving node i choosing the path of commodity k
w_k = The weight of commodity k.

and our MILP model is formulated as follows:

$$\text{Maximize} \quad \sum_{i \in N_s} \sum_k x_{iTJ^*k} \tag{10}$$

Subject to:

$$\sum_k b_{ik} = b_i \quad \forall i \in N \tag{11}$$

$$\sum_{j|j=J' \text{ or } (i_t, j_{t+tr_{ij}}) \in A_T} x_{itjk} = b_{ik} \quad \forall i \in N_d, t=1, \forall k \tag{12}$$

$$\sum_{j|(i_t, j_{t+tr_{ij}}) \in A_T} x_{itjk} = b_i \quad \forall i \in N_s, t=1, \forall k \tag{13}$$

$$\sum_{j|(i_t, j_{t+tr_{ij}}) \in A_T} x_{itjk} - \sum_{j|(j_{t-tr_{ji}}, i_t) \in A_T} x_{j(t-tr_{ji})ik} = 0 \quad \forall i \in N, t \in \{2, \ldots, T-1\}, \forall k \tag{14}$$

$$x_{itJ^*k} - \sum_{j|(j_{t-tr_{ji}}, i_t) \in A_T} x_{j(t-tr_{ji})ik} = 0 \quad \forall i \in N_s, t=T, \forall k \tag{15}$$

$$\sum_k x_{itjk} \leq UA_{ij} \quad \forall (i,j) \in A, t \in \{1, \ldots, T\} \tag{16}$$

$$\sum_k x_{itik} \leq UN_i \quad \forall i \in N, t \in \{1, \ldots, T\} \tag{17}$$

$$\sum_{j|(i,j) \in A} y_{ijk} - \sum_{j|(j,i) \in A} y_{jik} = 1 \quad \forall i \in N_d, \forall k, \text{ if } i \text{ is the source of commodity } k \tag{18}$$

$$\sum_{j|(i,j) \in A} y_{ijk} - \sum_{j|(j,i) \in A} y_{jik} = 0 \quad \forall i \in N, \forall k, \text{ if } i \text{ is not the source or the sink of commodity } k \tag{19}$$

$$\sum_{j|(i,j) \in A} y_{ijk} - \sum_{j|(j,i) \in A} y_{jik} = -1 \quad \forall i \in N_s, \forall k, \text{ if } i \text{ is the sink of commodity } k \tag{20}$$

$$x_{itjk} \leq UA_{ij} \times y_{ijk} \quad \forall (i,j) \in A, t \in \{1, \ldots, T\}, \forall k \tag{21}$$

$$x_{itjk} \geq 0 \quad \forall (i,j) \in A, t \in \{1, \ldots, T\}, \forall k \tag{22}$$

$$x_{itik} \geq 0 \quad \forall i \in N, t \in \{1, \ldots, T\}, \forall k \tag{23}$$

$$x_{i1J'k} \geq 0 \quad \forall i \in N_d, \forall k \tag{24}$$

$$x_{iTJ^*k} \geq 0 \quad \forall i \in N_s, \forall k \tag{25}$$

$$y_{ijk} \in (0,1) \quad \forall (i,j) \in A, t \in \{1, \ldots, T\}, \forall k \tag{26}$$

Module 4: Heuristic Solution Algorithm

Since the multi-commodity MILP model runs very slow for large scale problems due to the huge number of variables and constraints, we came up with a heuristic algorithm to get a good solution in the quickest time. The proposed heuristic algorithm consists of five steps, and it is designed to handle multiple evacuation routes from each of the impact nodes to safe nodes. In evacuating people from nodes, it is necessary that the nodes with the highest number of evacuees and the earliest impact time be evacuated first, followed by the nodes with the second highest supply of evacuees and second earliest impact time and so on. Therefore, the first step is to prioritize the impact nodes. Once the prioritization is complete, depending on the supply and the impact time, the number of feasible routes from the impact nodes to the safe nodes is identified in the second step. Following this, in the third step, a set of unique evacuation routes are generated for each impact node using the shortest path algorithm []. These routes are based on the shortest time it requires to reach safe nodes. The forth step is to create a time expanded network for each of the routes and maximize the flow of evacuees to safe nodes within planning time horizon. With every flow, the arc capacities and node supplies are updated and the step is repeated for the next route. The last or fifth step involves obtaining the evacuation routes and the schedule from the solution generated.

4. Numerical Results We ran both multi-commodity model and heuristic model for six different networks to compare the solution time and number of evacuees between these two methods. Table3 shows the specification of each problem along with the numerical results. Optimization models are formulated in GAMS 22.6 and solved using CPLEX while the heuristic algorithms are written in C++. As can be seen from this table, the heuristic method runs faster than MILP model while maintaining the same evacuation ratio..

ID	Number of Nodes	Evacuation (%)		Time (Sec.)	
		Heuristic	MILP(CPLEX)	Heuristic	MILP(CPLEX)
1	3	100%	100%	0.047	0.328
2	9	100%	100%	0.406	0.859
3	18	100%	100%	6.094	8.828
4	39	90.79%	N/A	897.967	Out of Memory
5	49	80.01%	N/A	955.109	Out of Memory
6	60	100%	N/A	3190.72	Out of Memory

Table 3: Numerical Results

5. Conclusion

We have developed an optimization framework for a short notice evacuation routing and scheduling. Our four-step approach includes two LP models for finding a lower bound on the number of evacuees and an upper bound on the evacuation time. Then a MILP model was presented for finding efficient evacuation rote selection and schedules. Finally, a heuristic algorithm was developed to expedite the solution process.

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