Numerical Simulation of Surge Overflow at a Levee Using Shallow Water SPH Method

Xin Rao[1], Lin Li[2], and Farshad Amini[3]

Abstract - In this paper, a purely Lagrangian and meshless approach, the smoothed particle hydrodynamics (SPH) method, was used to explore its ability to solve shallow water equation (SWE), which was originally used in flat landform condition. A method was developed to solve complicated landform problem in a levee, and a special technique was introduced to deal with the moving flow front. The results show that the SPH method for shallow water flows can provide satisfactory solution to surge overflow condition for a levee.

Key words--SPH method, shallow water flows, surge overflow, levee

I. INTRODUCTION

The smoothed particle hydrodynamics (SPH) method was originally introduced to simulate astrophysical problems. By the end of the 1980s and in the early 1990s, the SPH method was widely used in conjunction some other meshless methods especially in the simulation of high speed impacts and metal forming processes [1].

The SPH method is a meshless Lagrangian method that uses a pseudo-particle interpolation algorithm to calculate smooth field variables. Each pseudo-particle has a mass, Lagrangian position, Lagrangian velocity, and internal energy. Other quantities can be derived by interpolation or developed from constitutive relations. The pseudo-particles move with the velocity of the continuum, but are not associated with a grid and consequently do not have fixed connectivity.

Free surface flows are often numerically solved using Eulerian approaches such as the finite volume or finite element methods. However, there are difficulties in simulating moving domain, such as the wetting–drying phenomena particularly in flood simulations. SPH is one of alternative methods to address the moving domain problems.

Shallow water equations (SWE) have been solved using the SPH method [2]. Raidh (2005) method was originally used in flat landform condition. How to apply the SPH in a complicated landform problem, such as levee, is still unknown. The purpose of this paper is to improve SPH for solving SWE, especially for the case of landform and moving flow front.

II. THEORY

A. Hydraulics Formula

The inviscid SWE in the non-conservative form neglecting the bed slope and friction terms is written as:

$$\frac{Dh}{Dt} + h\nabla\vec{u} = 0 \quad (1)$$

$$\frac{D\vec{u}}{Dt} + g\nabla h = 0 \quad (2)$$

[1]Xin Rao is with the Department of Civil and Environmental Engineering, Jackson State University, Jackson, MS, USA, 39217 on leave from the College of Water Conservancy and Hydropower Engineering, Hohai University, Nanjing, China 210098 (e-mail: 175824262@qq.com).

[2]Lin Li, PhD, PE, F.ASCE is with the Department of Civil and Environmental Engineering, Jackson State University, Jackson, MS, USA, 39217 (e-mail: lin.li@jsums.edu).

[3]Farshad Amini, PhD, PE, is with the Department of Civil and Environmental Engineering, Jackson State University, Jackson, MS, USA, 39217 (corresponding author: 1-601-9793913; fax: 1-601-979-3238; e-mail: famini@jsums.edu)

where h, u and g are respectively water height, depth-averaged velocity, and gravity. D/Dt refers to the total derivative.

The SPH discrete form of the momentum equations (2) is obtained as:

$$\frac{D\vec{u}_i}{Dt} = -\sum_{j=1}^{N} v_j \left(g(h_j + h_i) + \Pi_{ij}\right) \nabla w(x_i - x_j) \qquad (3)$$

where h_i is the nodal water height of particle i.

In order to avoid the oscillations near shocks and interpenetration of particles, artificial viscosity was proposed by Monaghan and is given by [4]

$$\Pi_{ij} = \begin{cases} -\alpha_{ij}\mu_{ij} + \beta \tilde{c}_{ij}\mu_{ij}^2 & (u_i - u_j)(x_i - x_j) < 0 \\ 0 & elsewhere \end{cases} \qquad (4)$$

where α and β are constants, $\tilde{C}_{ij}$ is the average of wave speed associated with particles i and j and $u_{ij} = \left(u_i - u_j\right)\cdot\left(x_i - x_j\right) = \left(\left(x_i - x_j\right)^2 + \varepsilon^2\right)$. Typically, the parameters α and β are often taken as 1.0 [3][4].

The continuity equation is implicitly satisfied since a Lagrangian kinematic approach is used, i.e. the particle masses are conserved. Water depth h can be computed using an SPH approximation as:

$$h^a(x_i) = \sum_{j=1}^{N} v_j h_j w(x - x_j) = \sum_{j=1}^{N} m_j w(x - x_j) \qquad (5)$$

where m_j is the mass of particle j. The superscript *a* indicates an approximate value.

B. Boundary Treatment

One of major boundary treatment methods in SPH is ghost (virtual) particle technique [5]. The principle of dealing with boundary is by making the kernel to have a compact support, i.e., full of particles in the smoothing circle (Fig. 1). Near the boundary, the kernel does not have complete smoothing circle. Symmetrical ghost particles needed to be added to make the smoothing circle completely full. As the water boundary is moving flow front, the ghost particles method is used in this study by adding additional virtual particles so that the smoothing circle is completely full. Depending on the boundary conditions, ghost particles were assigned with different properties in velocity, mass, and volume.

(3)

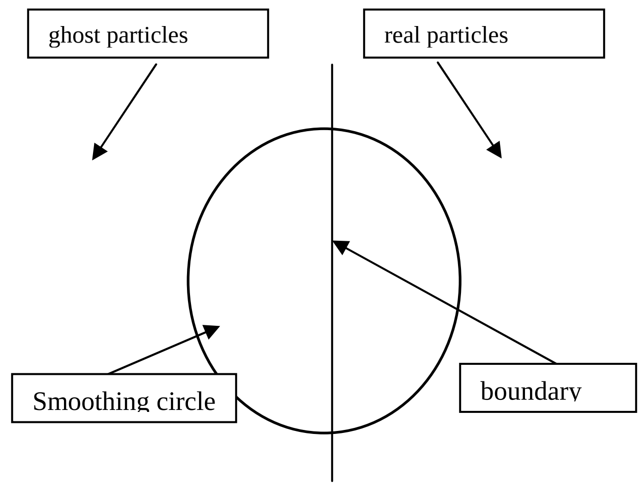


Figure 1. Illustration of ghost particles method

In this study, ghost particles were assigned in the dealing with moving flow front boundary. Negative water depth and velocity of real particles were used for the ghost particles to keep the pressure and velocity of the boundary equal to zero.

C. Slope Landform

The slope is divided into many fixed particles. Each particle has its own properties: mass, height, volume, velocity, and acceleration. The height is the height of slope. If the heights are different from others, the masses are different homologically. The velocity and acceleration of these fixed particles are equal to zero forever. The properties of fixed slope particles remain unchanged in the computation.

D. Nearest Neighboring Particles Searching Algorithm

In SPH, the distance between the particles varies in time. Thus, at each time step, the nearest neighboring particles for every specified particle have to be found. Usually, algorithm for one dimensional flow problems is available. The algorithm for two dimensional flow problems can be founded with linked-list algorithm [5].

III. RESULT

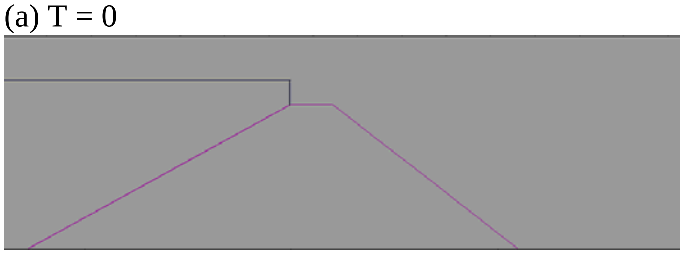
In this research project, a one-dimensional overflow model at a levee was used to investigate the applicability of SPH method for a surge overflow.

The height of the levee is 3 m and the crest width is 2 m. The water-side slope is 4.25H:1V and the land-side slope is 3H:1V. An initial water level of 3.3 m is specified creating 0.3 m of surge overflow. A constant inflow discharge of 0.05 m^3/s is specified on the left boundary of the rectangular solution domain. The initial velocity was set as 0.1m/s in this example.

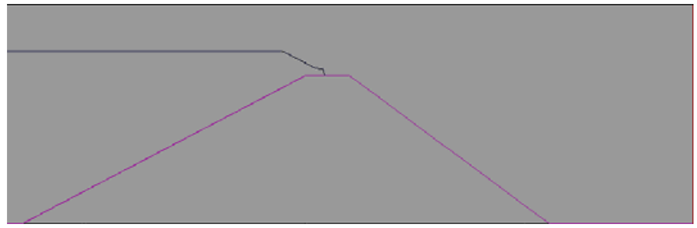
Fig. 2. shows the instantaneous free surface profiles at ten different time instants. At time T = 0, the water is confined to the left hand side of the levee but the water level is 0.3 m above the crest top. Due to the presence of constant inflow, the flow is pushed over the top of the levee crest and the water level at the water-side slope of the levee is rising during the initial state of simulation. On the land-side slope of the levee, the water flow accelerates rapidly due to gravitational effect. The water depth near the toe of the levee is considerably shallower than that on the levee crest as the overtopping water flow velocity increases along the levee surface under the effect of gravity.

The SPH results will be compared with the analytical solutions, full-scale laboratory tests and other numerical solutions in the future.

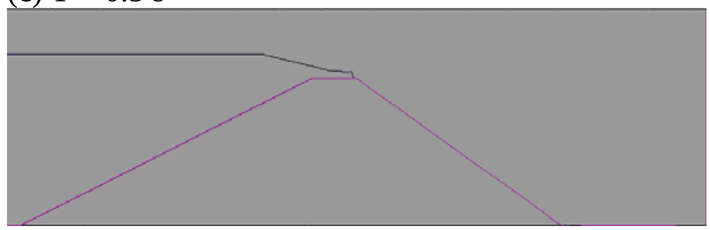
(a) T = 0



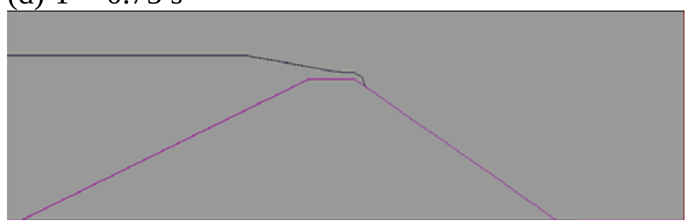
(b) T = 0.25 s



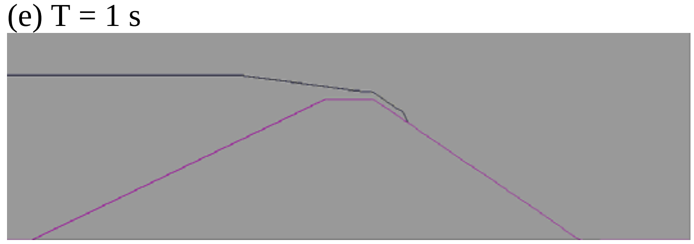
(c) T = 0.5 s



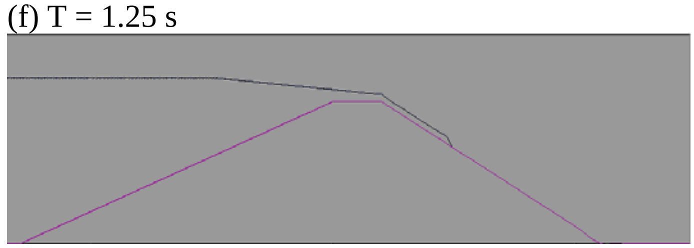
(d) T = 0.75 s



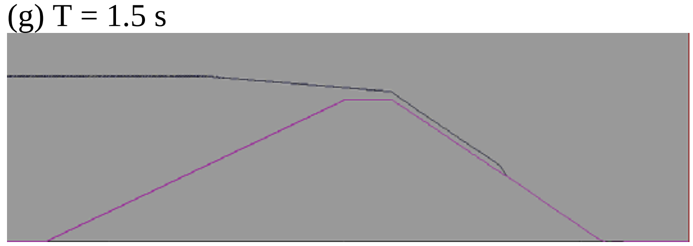
(e) T = 1 s



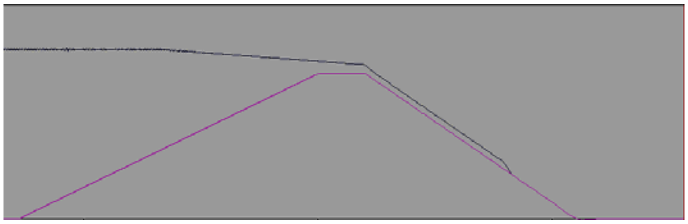
(f) T = 1.25 s



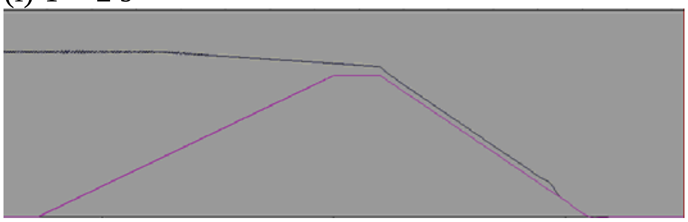
(g) T = 1.5 s



(h) T = 1.75 s



(i) T = 2 s



(j) T = 3 s

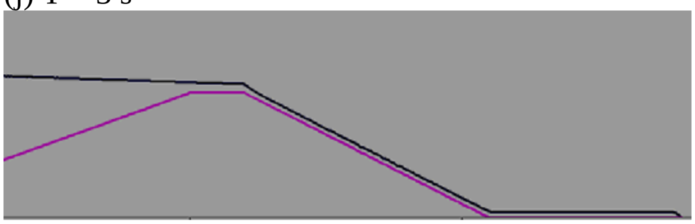


Figure 2. Free surface elevation contour at time: (a) T = 0; (b) T = 0.25 s; (c) T = 0.5 s; (d) T = 0.75 s; (e) T = 1 s; (f) T = 1.25 s; (g) T = 1.5 s; (h) T = 1.75 s; (i) T = 2 s; and (j) T = 3 s

IV. CONCLUSIONS

SPH can simulate free surface overflow problems. But the method has some problems when dealing with SWE. In the present work, a simple technique was used to solve the complicated landform problems and a method was carried out to satisfy the moving flow front conditions. In the future, we plan to solve the remaining problems so that the SPH method can be used in real engineering environment.

ACKNOWLEDGMENT

This research project is funded by the Department of Homeland Security (DHS) through Oak Ridge National Laboratory. This support is gratefully acknowledged. The conclusions in this paper are solely those of the authors and do not necessarily reflect the opinions or policies of DHS. Endorsement by DHS is not implied and should not be assumed.

REFERENCES

[1] Gingold R.A. and Monaghan J.J, “Smoothed Particle Hydrodynamics: Theory and Application to Non-spherical stars”, Monthly Notices of the Royal Astronomical Society, vol.181, pp. 375-389, 1977.

[2] Raidh, A., “A stabilized SPH method for inviscid shallow water flows”, International Journal for Numerical Methods in Fluids, vol. 47, no. 2, 2005.

[3] Lattanzio, J. C., Monaghan J.J., Pongracic, H. and Schwartz M.P., “Controlling penetration”, SIAM Journal on Scientific and Statistical Computing, vol.7, no.2, pp.591-598,1986.

[4] Monaghan J.J., “On the problem of penetration in particle methods”, Journal of Computational physics, vol. 82, pp. 1-15, 1986.

[5] Liu M. B., Liu G.R. and Lam K.Y., “Investigations into water mitigations using a meshless particle method”, Shock Waves, vol.12, no.3, pp. 181-195, 2002.