Modeling of Surge Overtopping and Hydraulic Erosion of an Earthen Levee Using Smoothed Particle Hydrodynamics

X. Rao
Jackson State University, Jackson, MS, USA, on leave from the College of Water Conservancy and Hydropower Engineering, Hohai University, Nanjing, China (175824262@qq.com).
L. Li
Jackson State University, Jackson, MS, USA (lin.li@jsums.edu)
F. Amini
Jackson State University, Jackson, MS, USA (famini@jsums.edu)

ABSTRACT Smoothed particle hydrodynamics (SPH) method was combined with a physically-based erosion model adopted from an Eulerian approach to explore its ability to solve shallow water equation during surge overtopping and corresponding hydraulic erosion of an earthen levee. A method was developed to solve complicated landform problem in a levee, and a special technique was introduced to deal with the moving flow front. Results show that the SPH method for shallow water flows and hydraulic erosion can provide practical solution to surge overtopping and erosion for a levee.

INTRODUCTION

The smoothed particle hydrodynamics (SPH) method, a Lagrangian and meshless numerical method, was first introduced by Gingold and Monaghan (1977) to simulate astrophysical problems. Since the motion of heavenly bodies is much alike the flow and gas, it is used to solve fluid and solid mechanics problems, particularly in the simulation of high speed impacts and metal forming processes (Liu and Liu 2003).

SPH method includes two steps: description and approximation of particles. In the description step, the domain is divided into many particles. The discrete particles have their own properties, etc. mass, position, velocity, volume. Thus, the particles can describe and represent the domain, and they will move according to the law of the governing equation. In the second step, a suitable approximate method is chosen to obtain discrete SPH equation, to calculate the new properties every time step based on the initial ones.

The SPH method has been recently used in more and more fields, and its wide application promotes the further development of SPH. The SPH is now used to solve free surface instead of Eulerian approaches such as the finite volume or finite element methods. The SPH discrete form of shallow water equation in one or two dimensions was used to simulate many free surface flow problems (Raidh 2005). A new physically-based erosion SPH model was introduced to couple erosion simulation with Lagrangian approach (Kristof et al. 2009). These progresses of SPH make this method more competitive and adaptive to real engineering case.

Levee breach is often numerically solved using Eulerian approaches since the case has complicated landform and erosion process. Because of the wetting–drying flood phenomena, SPH is a good choice. In this paper, SPH was developed to simulate surge overflow and corresponding hydraulic erosion.

THEORY

Hydraulics Formulation

The inviscid SWE in the non-conservative form neglecting the bed slope and friction terms is written as:

$$\frac{Dh}{Dt} + h\nabla\bar{u} = 0 \quad (1)$$

$$\frac{D\bar{u}}{Dt} + g\nabla h = 0 \quad (2)$$

where h, u and g are respectively water height, depth-averaged velocity, and gravity. D/Dt refers to the total derivative.

The SPH discrete form of the momentum equations (2) is obtained as (Raidh 2005):

$$\frac{D\bar{u}_i}{Dt} = -\sum_{j=1}^{N} v_j (g(h_j + h_i) + \Pi_{ij}) \nabla w(x_i - x_j) \quad (3)$$

where h_i is the nodal water height of particle i.

In order to avoid the oscillations near shocks and interpenetration of particles, artificial viscosity was proposed by Monaghan (1986) and is written as:

$$\Pi_{ij} = \begin{cases} -\alpha_{ij}\tilde{c}_{ij}\mu_{ij} + \beta\mu_{ij}^2 & (u_i - u_j)(x_i - x_j) < 0 \\ 0 & \text{elsewhere} \end{cases} \quad (4)$$

Where α and β are constants, $\tilde{c}_{ij}$ is the average of wave speed associated with particles i and j and $\mu_{ij} = \ell(u_i - u_j)(x_i - x_j)$. The parameters α and β are often taken as 1.0 (Rao et al. 2009).

The continuity equation is implicitly satisfied since a Lagrangian kinematics approach is used, i.e. the particle masses are conserved. Water depth h can be computed using an SPH approximation as:

$$h(x_i) = \sum_{j=1}^{N} v_j h_j w(x - x_j) = \sum_{j=1}^{N} m_j w(x - x_j) \quad (5)$$

Where m_j is the mass of particle j.

Erosion

Shear stress is obtained as (Kristof et al. 2009):

$$\tau = k\theta^n \quad (6)$$

Where τ is the shear stress, k is the shear stress constant, θ is the shear rate and n is the flow behaviour index.

The shear rate can be approximated as:

$$\theta = v_{rel}/l \quad (7)$$

Where l is the distance between fluid and boundary.

The erosion rate is formulated as:

$$\varepsilon = k_\varepsilon(\tau - \tau_c) \quad (8)$$

Where k_ε is the erosion strength and τ_c is the critical shear stress.

The SPH discrete form of mass change is expressed as (Kristof et al. 2009):

$$\frac{dM_b}{dt} = -\sum_j L_b^2 \varepsilon(j) \quad (9)$$

Boundary

One of major boundary treatment methods in SPH is ghost (virtual) particle technique (Liu and Liu. 2003). The principle of dealing with boundary is by making the kernel to have a compact support, i.e., full of particles in the smoothing circle (Fig. 1). Near the boundary, the kernel does not have complete smoothing circle. Symmetrical ghost particles needed to be added to make the smoothing circle completely full. Depending on the boundary conditions, ghost particles were assigned with different properties in velocity, mass, and volume. In this study, ghost particles were assigned in the dealing with moving flow front boundary. Negative water depth and velocity of real particles were used for the ghost particles to keep the pressure and velocity of the boundary equal to zero.

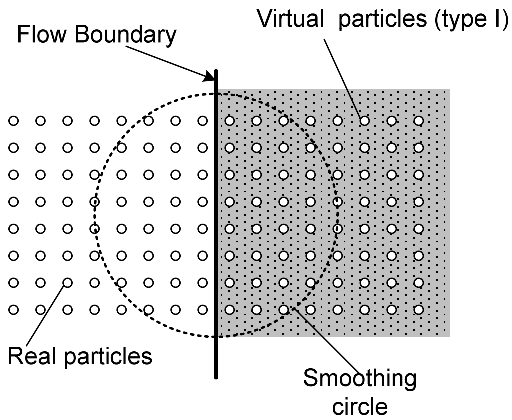

Fig. 1. Illustration of ghost particles method

If the boundary is solid, another type of virtual particles (type II) would be set up on the boundary. When the real particles approach the solid boundary, a sufficiently strong force is produced to avoid interpenetration of particles. This force would act along the centreline between virtual and real particles, and point to the real particles (Fig. 2).

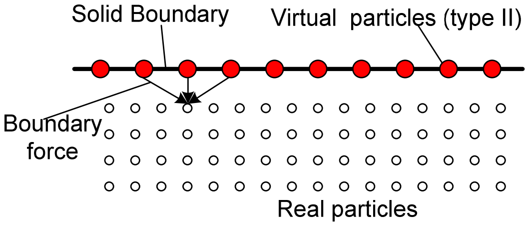

Fig. 2. Illustration of type II virtual particles on solid boundary

Slope Landform

The bed slope was divided into many fixed particles. Each particle has its own properties: mass, height, volume, velocity, acceleration. The height is the height of slope. If the heights are different from others, the masses are different homologically. The velocity and acceleration of these fixed particles is equal to zero forever. These fixed particles were placed on the boundary of the landform (Fig. 3). The mass of these particles was used to represent the erosion in every time step. The decreased value is the eroded mass.

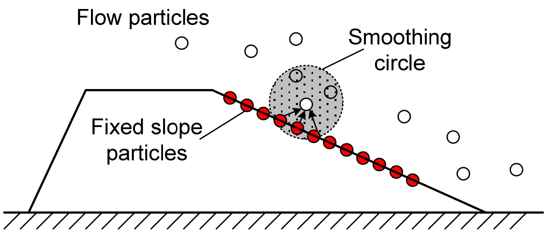

Fig. 3. Setting up of slope particles

Nearest Neighbouring Particles Searching Algorithm

In SPH, the distance between the particles varies in time. Thus, at each time step, the nearest neighbouring particles for every specified particle have to be found. All-pair search algorithm is usually used for 1-D problems, and linked-list algorithm is used for 2-D problems (Rao et al. 2009).

RESULTS

In this study, a one-dimensional conceptual model at a levee was used to investigate the applicability of SPH method for surge overflow and hydraulic erosion process. The height of the levee is 3 m and the crest width is 2 m. The water-side slope is 4.25H:1V and the land-side slope is 3H:1V. An initial water level of 3.3 m is specified creating 0.3 m of surge overflow. A constant inflow discharge of 0.05 m^3/s is specified on the left boundary of the rectangular solution domain.

Fig. 4 shows the instantaneous free surface profiles and hydraulic eroded levee profile at different time instants under surge overflow condition. To illustrate the hydraulic erosion process, relatively high erosion parameters were used to avoid the concussion near the boundary. When the surge overflow passed the crest and landed on the land-side slope, the accelerated water flow caused higher erosion stress on the landside slope. When the erosion stress exceeded the critical shear stress, erosion occurred on the slope, as shown in Fig. 4b. The erosion extended from the landside slope to the top of the crest. When the top of the crest was eroded, the crest and land-side slope of the levee during the levee breech process were continuously eroded. After T = 3.25 s, the levee breach occurred.

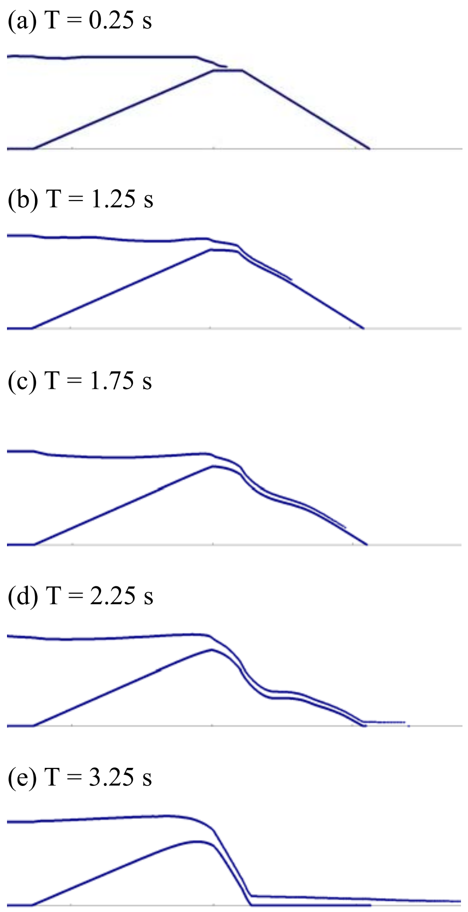

Fig.4. Levee breach at various time: (a) T = 0.25 s; (b) T = 1.25 s; (c) T = 1.75 s; (d) T = 2.25 s; and (e) T = 3.25 s

CONCLUSION

In this study, the SPH was modified to solve levee breach under surge overflow and related hydraulic erosion condition. New techniques were used to solve the combined surge overflow and hydraulic erosion during the levee breach process. Parametric study will be conducted in future work to study the most important parameter for the levee breach under surge overflow condition.

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