Overland flow modeling using two-dimensional shallow water equations

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Abstract

This paper presents development and application of a two-dimensional (2D) physically based numerical model for simulation of overland flow resulting from rainfall over agricultural watersheds. The model is based on 2D shallow water equations coupled with the Green-Ampt method for infiltration. The rainfall intensity and infiltration rate are directly inserted into the shallow water equations as source and sink terms. The model allows simulation of Hortonian overland flow and the infiltration process during complex storm events recorded by multiple rain gauges. The model is capable of considering the spatial variability of rainfall, soil characteristics and land-use. The spatially varying soil characteristics such as hydraulic conductivity, effective porosity, soil suction head and soil moisture are considered by assigning appropriate values using a soil map. The effect of land-use is considered by using spatially varying Manning's friction coefficient in the source terms of momentum equations. The potential evapotranspiration is not considered in this model. A well-balanced, second order accurate, central upwind, explicit and positivity preserving scheme is used for the solution of the governing equations. The proposed numerical model is calibrated and validated using four field scale rainfall-runoff experiments conducted at Niger (West Africa) in 1994. The simulation results indicate a good match between the model predictions and experimental observations. Finally, the model is applied to the real world scenario in the Goodwin Creek watershed in Mississippi. The runoff recorded at the outlet of the watershed as a result of two storm events is compared with the simulations and a close match is observed. The field experimental validation and real life simulations confirm that the proposed model is capable of simulating the overland flow to a good accuracy and can be applied for the solution of real time flash floods resulting from short-term heavy rainfall events.

Key words: Overland flow modeling; Shallow water equations; Central upwind schemes; Rainfall-runoff modeling; Infiltration rate.

1. Introduction

Heavy storm events can trigger devastating floods that may create havoc in residential areas in terms of loss of life, property damage, and agricultural damage and so on. Flash floods generated from overland flow can cause many deaths in mountainous regions where flood waves may hit without warning. At the beginning of a rainfall event, depending upon the type of soil and antecedent moisture conditions, some portion of the rains is lost by infiltration and evaporation. However, when the soil becomes saturated, the water starts flowing over the land and turns into overland flow rapidly as upland areas provide significant a amount of water. Generally, the rainfall-runoff modeling is performed using the distributed hydrological models, such as SWAT, AnnAGNPS, WEAP, etc. These models can simulate flood runoff to a reasonable accuracy on a larger temporal and spatial scale. However, the inundation map, arrival time, water depth, water surface elevation and propagation velocities of flash floods in the floodplain are not possible to simulate using these hydrological models. Addressing these issues, this paper presents a two-dimensional (2D) numerical model which is capable of simulating flash floods caused by complex storm events over natural topography. This model gives inundation maps, flood arrival time, maximum water surface elevation and flood hydrographs at any point in time and space in the floodplain.

To the best of our knowledge, the essential originality of this research work is that it is one of the first to attempt to solve the 2D unsteady shallow water equation in the case of a spatially variable rainfall and soil infiltration parameters. Many previous studies of modeling 2D overland flow (Chow and Ben-Zvi, 1973; Hromadka et al., 1987; Zhang and Cundy, 1989; Tayfur et al., 1993) have limited their application to constant rainfall intensity and spatially constant infiltration rate. Esteves et al. (2000) modeled overland flow solving 2D shallow water equations using the MacCormack scheme and reproduced experimental data to a good accuracy. They considered a time varying rainfall intensity, but the rainfall intensity and infiltration were spatially constant. Kivva and Zheleznyak (2005) developed a physically valid mathematical model using shallow water equations for simulation of runoff from rainfall in small catchments. They validated their model against one-dimensional dam break flow over impermeable bottom and applied it to the Butenya River catchment. The model underestimated observed runoff in case of a compound rainfall event.

More recently, Cea et al. (2010) validated the 2D depth-averaged shallow water equation model for forecasting rainfall-runoff from precipitation data in an urban area. However, single-peaked small duration storms were used and the study was limited to a laboratory scale. Costabile et al. (2011) presented a comparative analysis of overland flow models and found that in ideal test cases of horizontal bottom, the diffusive and kinematic waves models can also produce similar results as fully dynamic models. But in test cases where the bottom topography is complicated, only dynamic models could produce good results for discharge and water depths.

In this paper, we have developed a 2D numerical model based on a central upwind second order accurate, well-balanced, positivity-preserving explicit scheme. Consequently, simulations can start on an initially dry surface and handle calculations on dry and wet areas allowing a more realistic prediction of the interaction between rainfall, overland flow and infiltration. Singh et al. (2010) rigorously tested this model against the analytical solutions of three established bench-mark test problems for subcritical, transcritical and supercritical flows on impermeable surfaces and found that simulations and the analytical solution matched. In this paper, we have calibrated the model based on the one field experimental test case and validated the model for three more field experimental test cases. Finally the model is applied to simulate two storm events in the Goodwin Creek watershed.

This paper is organized as follows: In Section 2, the mathematical formulation and the governing equations with rainfall and infiltration terms are presented. Section 3 presents the solution mythology and the treatment of sources and sink terms (bottom elevation gradient, bottom friction terms, rainfall and infiltration terms). Section 4 presents the calibration and validation of the proposed model based on the field experiments. In Section 5, the model application on a real world case is demonstrated. Section 6 gives the conclusions for this research work.

2. Governing equations

2.1 Shallow water equations with rainfall and infiltration terms

The conservative form of 2D shallow water equations, consisting of a continuity equation with rainfall and infiltration, and two momentum equations for depth-averaged free surface flows, are written as:

$$\begin{bmatrix} h \\ hu \\ hv \end{bmatrix}_t + \begin{bmatrix} hu \\ hu^2 + \frac{1}{2}gh^2 \\ huv \end{bmatrix}_x + \begin{bmatrix} hv \\ huv \\ hv^2 + \frac{1}{2}gh^2 \end{bmatrix}_y = \begin{bmatrix} R - I \\ -gh\frac{\partial B}{\partial x} - S_{fx} \\ -gh\frac{\partial B}{\partial y} - S_{fy} \end{bmatrix}, \qquad (1)$$

where x and y are Cartesian coordinates describing the horizontal plane, t is time, $h(x,y,t)$ is water depth, $u(x,y,t)$ and $v(x,y,t)$ are the two components of the depth-averaged velocities in x and y directions, respectively. $R(x,y,t)$ is the rainfall intensity and $I(x,y,t)$ is the infiltration rate. The gravitational acceleration is denoted by g. $B(x,y)$ is the bottom elevation function describing an arbitrary or natural bathymetry, S_{fx} and S_{fy} , the components of the non-linear bottom friction terms due to its roughness in x and y directions.

The water surface elevation is represented by $w(x,y,t)$ such that $w = h+B$ (see figure 1). The bottom elevation $B(x,y)$ is assumed not to change with time, i.e., fixed bed is considered in this model. By simply replacing water depth (h) by the difference of surface water elevation and bottom elevation ($w-B$), and doing simple algebraic manipulations, Eq.1 is rewritten in terms of $w(x,y,t)$.

$$\begin{bmatrix} w \\ hu \\ hv \end{bmatrix}_t + \begin{bmatrix} hu \\ \frac{(hu)^2}{w-B} + \frac{1}{2}g(w-B)^2 \\ \frac{(hu)\,(hv)}{w-B} \end{bmatrix}_x + \begin{bmatrix} hv \\ \frac{(hu)\,(hv)}{w-B} \\ \frac{(hv)^2}{w-B} + \frac{1}{2}g(w-B)^2 \end{bmatrix}_y = \begin{bmatrix} R - I \\ -g(w-B)\frac{\partial B}{\partial x} - S_{fx} \\ -g(w-B)\frac{\partial B}{\partial y} - S_{fy} \end{bmatrix}, \quad (2)$$

Eq.2 can be written in vector form as follows:

$$U_t + F(U,B)_x + G(U,B)_y = S(U,B), \qquad (3)$$

where *U, F(U,B), G(U,B)* and *S(U,B)* are the vectors of primitive variables and the fluxes in x and y direction and sources and sink, are defined as follows:

$$\left.\begin{aligned} & U = \begin{bmatrix} w \\ hu \\ hv \end{bmatrix},\; F(U,B) = \begin{bmatrix} hu \\ \dfrac{(hu)^2}{w-B} + \dfrac{1}{2} g (w-B)^2 \\ \dfrac{(hu)\,(hv)}{w-B} \end{bmatrix}, G(U,B) = \begin{bmatrix} hv \\ \dfrac{(hu)\,(hv)}{w-B} \\ \dfrac{(hv)^2}{w-B} + \dfrac{1}{2} g (w-B)^2 \end{bmatrix}, \\ & S(U,B) = \begin{bmatrix} R-I \\ -gh\dfrac{\partial B}{\partial x} - S_{fx} \\ -gh\dfrac{\partial B}{\partial y} - S_{fy} \end{bmatrix}. \end{aligned}\right\} \qquad (4)$$

2.2 Definition of sources and sinks

2.2.1 Rainfall intensity

The least distance method is used to assign the DEM cells to the nearest rain gauge located in the study area. Spatially and temporally varying rainfall intensity *R(x,y,t)* data is provided to the model from the rain gauge readings directly.

2.2.2 Infiltration rate

The infiltration rate *I(x,y,t)* is calculated using the Green-Ampt method (Green and Ampt 1911). This is the most widely used method for one-dimensional vertical movement of water into soil. The method is developed from integration of Darcy's law by assuming infiltration from a ponded layer into homogeneous soil at uniform antecedent moisture conditions (Chow et al., 1984). A wetting front, which is a sharp boundary between the saturated soil above, and the soil at antecedent moisture conditions below, penetrate into soil at depth L in time t since onset of infiltration. The variables required for calculation of infiltration rate by Green-Ampt are shown in Figure 2, in which the soil depth is plotted on the y-axis, and moisture content is ploted on the x-axis. L is the depth of the wetting front, h_0 is ponded depth above surface, θ_i is antecedent moisture content of the soil, η is saturated soil moisture content, θ_e is effective porosity and θ_r is residual moisture content. The ponded depth is generally neglected in Green-Ampt method, but in flash floods due to heavy storm events, the surface water depth (ponded depth) can be considerably high and hence cannot be neglected in the present context. The final equation for calculation of the infiltration rate in mm/hr by the Green-Ampt method is as follows:

$$I = \frac{\partial F}{\partial t} = \eta\ K_s \left(\left(h - \psi_f \right) \frac{\Delta \theta}{\eta\ F} + 1 \right) \tag{5}$$

where F is the total infiltration (m), K_s is the saturated hydraulic conductivity (m/s), η is the porosity (-), Ψ_f is the dry suction (m), $\Delta\theta$ is the difference of saturated soil moisture and the antecedent soil moisture (-). Brooks and Corey (1964) studied the variation of dry suction with moisture content for many soils in the laboratory, and concluded that Ψ_f can be expressed as a logarithmic function of effective saturation S_e. The effective saturation is the ratio of available moisture to the maximum possible moisture content of the soil and has a range of $0 \le S_e \le 1$.

$$S_e = \frac{\theta - \theta_r}{\eta - \theta_r} \tag{6}$$

The change in moisture content ($\Delta\theta$), as the wetting front passes, is derived from algebraic manipulation on Eq. 6.

$$\Delta\theta = (1 - S_e)\theta_e \tag{7}$$

2.2.3 Bottom friction

The bottom friction terms in Eq.1, S_{fx} and S_{fy}, are defined as follows:

$$S_{fx} = gu\sqrt{u^2 + v^2} / C^2, \tag{8}$$

$$S_{fy} = gv\sqrt{u^2 + v^2} / C^2. \tag{9}$$

where C is the Chezy coefficient and is calculated from $C = (h)^{1/6} / n$, in which n is the Manning's coefficient.

3. ***Numerical solution of governing equations***

3.1 Numerical scheme

Two-dimensional modeling of overland flow presents many challenges, such as mixed flow regimes, treatment of large source and sinks terms due to complex natural topography and wetting/drying fronts, etc. The classical conservative schemes for solving the shallow water equations employ Riemann solvers to calculate fluxes, which require opening of Riemann fan. In the present paper, we adopt a different approach, that uses a Godunov-type central upwind scheme (Kurganov and Petrova 2007), which is based on integration over the Riemann fan and is second order accurate in space. This scheme does not require characteristic decomposition and Riemann solver to calculate inter-cell fluxes. The positivity of the depth is guaranteed at the wet and dry interface by introducing a slope limiter and by keeping the courant number less than 0.25. The detailed information on this implementation can be found in Singh et al. 2011. To save space, in this paper only a brief explanation is provided.

Eq. 3, with the definitions of vectors given in Eq. 4, governs the motion of overland flow over complex natural terrain including rainfall and infiltration rates as source and sink terms. The discretization of these governing equations is based on a finite volume method. A staggered grid is used in which the primitive variables (*w*, *u* and *v*) and rainfall and infiltration are defined at the cell center, and the bottom elevation is defined at the cell corner as shown in Figure 3. The bottom is represented as a bilinear surface. In order to facilitate the direct input of a DEM dataset, for natural terrains, a uniform grid of size Δx in x-direction and Δy in y-direction is adopted. The (i^{th}, j^{th}) computational cell is defined as $\lfloor x_{i-1/2,j} x_{i+1/2,j}, y_{i,j-1/2}, y_{i,j+1/2} \rfloor$. Using Kurganov and Petrova (2007) scheme, Eq.3 can be solved as follows:

$$\frac{d}{dt}U_{i,j}(t) = -\frac{H^x_{i+\frac{1}{2},j}(t) - H^x_{i-\frac{1}{2},j}(t)}{\Delta x} - \frac{H^y_{i,j+\frac{1}{2}}(t) - H^y_{i,j-\frac{1}{2}}(t)}{\Delta y} + S_{ij}(t), \tag{10}$$

The central-upwind numerical fluxes H^x and H^y in x and y-directions respectively, are given below:

$$\left.\begin{aligned} H^x_{i+\frac{1}{2},j} &= \frac{a^+_{i+\frac{1}{2},j} f(U^E_{i,j}, B(x_{i+\frac{1}{2}}, y_j)) - a^-_{i+\frac{1}{2},j} f(U^W_{i+1,j}, B(x_{i+\frac{1}{2}}, y_j))}{a^+_{i+\frac{1}{2},j} - a^-_{i+\frac{1}{2},j}} + \frac{a^+_{i+\frac{1}{2},j} a^-_{i+\frac{1}{2},j}}{a^+_{i+\frac{1}{2},j} - a^-_{i+\frac{1}{2},j}} \left[U^W_{i+1,j} - U^E_{i,j}\right], \\ H^y_{i,j+\frac{1}{2}} &= \frac{b^+_{i,j+\frac{1}{2}} f(U^N_{i,j}, B(x_i, y_{j+\frac{1}{2}})) - b^-_{i,j+\frac{1}{2}} f(U^S_{i,j+1}, B(x_j, y_{j+\frac{1}{2}}))}{b^+_{i,j+\frac{1}{2}} - b^-_{i,j+\frac{1}{2}}} + \frac{b^+_{i,j+\frac{1}{2}} b^-_{i,j+\frac{1}{2}}}{b^+_{i,j+\frac{1}{2}} - b^-_{i,j+\frac{1}{2}}} \left[U^S_{i,j+1} - U^N_{i,j}\right] \end{aligned}\right\} \tag{11}$$

Where $a^{\pm}_{i\pm 1/2, j\pm 1/2}$ is the local one-sided speed of the wave propagation in x direction. The superscripted letters E ,W, N and S indicate the interpolated variable at east, west, north and south side of the cell. Interested readers are referred to Singh et al. (2010) and Kurganov and Petrova (2007) for detailed description of the solution scheme.

3.2 Solution of green-ampt equation:

Total infiltration in Eq. 5 is discretized using the forward Euler method as follows:

$$F_{i,j}^{n+1} = F_{i,j}^{n} + n\Delta t K_s \left(\left(h_{i,j} - \psi_f\right) \frac{\Delta\theta}{nF_{i,j}^{n}} + 1 \right) \quad (12)$$

The infiltration rate and rainfall intensity terms are implemented in the continuity equation of shallow water equations as follows:

$$w_{i,j}^{n+1} = w_{i,j}^{n} - \frac{\Delta t}{\Delta x}\left((H^x)^{(1)}_{i+1/2,j} - (H^x)^{(1)}_{i-1/2,j}\right) - \frac{\Delta t}{\Delta y}\left((H^y)^{(1)}_{i,j+1/2} - (H^y)^{(1)}_{i,j-1/2}\right) + \Delta t R_{i,j} - (F_{i,j}^{n+1} - F_{i,j}^{n}) \quad (13)$$

F_{ij}^{n+1} and F_{ij}^{n} are the cumulative infiltration at current and next time level. The difference (F_{ij}^{n+1} - F_{ij}^{n}) is the infiltration rate $\Delta t I_{ij}$ during the time step. The method of successive substitution is used for cumulative infiltration F_{ij}^{n} at the first time step with $F_{ij}^{n} = K_s t$ as the first trial. There is published literature for the value of K_s, η, Ψ_f , θ_e for various soil classes. (Chow et al., 1984). The value of S_e can be found by calibration. If any cell in the DEM is dry, then the infiltration terms are set to zero in Eq. 13. However, the computation of movement of the wetting front in the soil layer is continuously updated using Eq. 12.

3.3 Implementation of bottom elevation gradient

The discretization of source terms due to the bottom elevation gradient is of crucial importance as the non-zero component of the flux terms needs to be balanced with the bottom elevation source terms in case of steady state solutions. The *w(x,y,t)* remains constant for the steady state condition. The source terms due to bottom elevation gradient in *x*-direction $S_{ij}^{(2)}$ is discretized as follows:

$$\left.\begin{aligned}
\overline{S}_{i,j}^{(2)}(t) &\approx -g\,\frac{B(x_{i+\frac{1}{2}}, y_j) - B(x_{i-\frac{1}{2}}, y_j)}{\Delta x} \cdot \frac{(w_{i,j}^{E} - B(x_{i+\frac{1}{2}}, y_j)) + (w_{i,j}^{W} - B(x_{i-\frac{1}{2}}, y_j))}{2} \;, \\
\overline{S}_{i,j}^{(3)}(t) &\approx -g\,\frac{B(x_i, y_{j+\frac{1}{2}}) - B(x_i, y_{j-\frac{1}{2}})}{\Delta y} \cdot \frac{(w_{i,j}^{N} - B(x_i, y_{j+\frac{1}{2}})) + (w_{i,j}^{S} - B(x_i, y_{j-\frac{1}{2}}))}{2}
\end{aligned}\right\} \qquad (14)$$

In order to guarantee the positivity of the flow depth at the cell center, the slope of newly reconstructed primitive variables is compared with the slope of the bottom elevation. If the reconstructed values of primitive variables at the mid-points of the cell interface is less than that of bottom elevation at the cell interface mid-points, the slope is adjusted explicitly equal to the bottom slope in order to assure that the flow depth remains always positive. Like other explicit schemes, this scheme is subjected to the Courant-Friedrichs-Lewy (CFL) condition for stability. The scheme is unconditionally stable if the CFL number for 2D scheme is 0.25 as suggested in the original scheme (Kurganov and Petrova 2007).

3.4 Implementation of friction terms

The friction term in *x*-direction S_{fx} is implemented in momentum equation for *x*-direction as follows:

$$\left.\begin{aligned}(hu)_{i,j}^{n+1} = \frac{(hu)_{i,j}^{n}}{(1+\alpha)_{i,j}^{n}} - \frac{\Delta t}{\Delta x}\frac{1}{(1+\alpha)_{i,j}}\left((H^x)_{i+1/2,j}^{(2)} - (H^x)_{i-1/2,j}^{(2)}\right) \\ - \frac{\Delta t}{\Delta y}\frac{1}{(1+\alpha)_{i,j}}\left((H^y)_{i,j+1/2}^{(2)} - (H^y)_{i,j-1/2}^{(2)}\right) - \frac{\Delta t\, S_{i,j}^{(2)}}{(1+\alpha)_{i,j}}\end{aligned}\right\} \tag{15a}$$

$$\left.\begin{aligned}(hv)_{i,j}^{n+1} = \frac{(hv)_{i,j}^{n}}{(1+\alpha)_{i,j}^{n}} - \frac{\Delta t}{\Delta x}\frac{1}{(1+\alpha)_{i,j}}\left((H^x)_{i+1/2,j}^{(3)} - (H^x)_{i-1/2,j}^{(3)}\right) \\ - \frac{\Delta t}{\Delta y}\frac{1}{(1+\alpha)_{i,j}}\left((H^y)_{i,j+1/2}^{(3)} - (H^y)_{i,j-1/2}^{(3)}\right) - \frac{\Delta t\, S_{i,j}^{(3)}}{(1+\alpha)_{i,j}}\end{aligned}\right\} \tag{15b}$$

Where, $\alpha = \dfrac{\Delta t\, g\sqrt{u^2+v^2}}{h_{i,j}C^2}$, is obtained by doing some algebraic manipulation on S_f term.

4. *Model calibration and validation*

The calibration and validation of the model is performed using the field experimental studies reported by Esteves et al. (2000). This field experiment was performed in Niger (West Africa). The layout of the experimental field is shown in Figure 4. The upstream and the side walls were non porous and downstream side is the outlet to collect surface runoff discharge. The experimental field was 14.25 m long and 5 m wide with crusted soil surface without vegetation. The average slope of the field in *x* and *y* direction was 0.0196 and 0.064 respectively. The soil texture was loamy sand. A total of four storm events (refer to Table 1) were simulated using this model. The duration of storm events ranged from 1500 seconds to 5000 seconds. The rainfall was measured with an electronic tipping bucket recording rain gauge, each tipping corresponding to 0.5 mm of rain depth. The discharge from the plot was measured at its outlet during the selected rainfall events. The downstream side AB was considered as the outlet and the water was collected in a gutter and flowed through a 20^0 V-notch weir.

Table 1

Event No.	Date	Initial soil moisture	Total rainfall depth (mm)	Max. rainfall intensity (mm/h)	Rainfall duration (s)
1	10th August 94	0.106	13.0	75.0	1520
2	7th August 94	0.100	31.0	200.0	2055
3	25th August 94	0.085	24.5	138.0	2446
4	4th September 94	0.082	62.5	138.46	12581

The first event is a single-peak storm used for calibration of the model. The remaining three events are multi-peak storms which are used to validate the model. The model needs six parameters, namely Manning's coefficient, hydraulic conductivity, porosity, effective porosity, effective saturation, and wetting front soil suction head. These initial estimates of these parameters are selected from Chow et al. (1988) for loamy sand type textured soils. The final values, for which the model produced the closest match with the experimental data, are selected for the simulation of the remaining three events. The selected values of these parameters are listed in Table 2.

To solve Eq. 12, the initial value of cumulative infiltration for the first time step must be known. The method of successive substitution is used for estimating this initial value. As suggested in Chow et al. (1988), the first trial value for successive substitution method is estimated as $F= K\Delta t$, where Δt is 0.001 s.

Table 2

Soil Type	Loamy Sand
Manning's n (s $m^{-1/3}$)	0.02
Hydraulic Conductivity (cm/h)	2.99
Porosity	0.437
Effective Porosity	0.401
Effective Saturation	0.30
Wetting front soil suction head (cm)	6.13

The computational conditions for this field test are as follows: The $\Delta x = \Delta y = 0.25$ m. The time step was governed by the CFL criteria while the courant number kept equal to 0.25. The Mannings's n is selected as 0.02 $m^{-1/3}s$ in order to find the best match between the observed and simulated values of runoff. The wall-type boundary condition is imposed along the non-porous sides whereas open-type boundary condition is imposed along the downstream side of the domain. The discharge is collected by summing up the surface runoff discharge along the downstream side of the domain. The discharge is then divided by the total area in order to convert it into a runoff value in mm/hr.

Since initial soil moisture is available for these field experimental cases, $\Delta\theta$ is calculated directly and an effective saturation parameter (in Eq.15) is not required. The comparison of experimental runoff generated from the first storm event with the model simulations using the finally selected parameters is shown in Figure 5. It is observed that the simulated runoff hydrograph is matching closely with the field observations. The peak runoff rate simulated by this model is closer to the field results than the result produced by the model based on the Mac Cormack scheme (Esteves et al. 2000). The base of the hydrograph is in complete agreement with the field observations.

The comparison of the simulated runoff hydrographs and field-observed hydrograph for storm nos 2, 3 and 4 are shown in Figures 6, 7 and 8. These three storms are multi-peak complex storms. It can be observed by comparison that the model is capable of producing comparable results in case of complex storms, too. All the runoff peaks and the time of their occurrence are reproduced well by the model. The shape of the hydrograph depends upon many watershed characteristics in addition to the shape of the rainfall hyetograph. Here the watershed is almost a plain area with mild slopes. The rainfall hyetographs are also superimposed on the hydrographs for all the storm events. The shapes of the three simulated hydrographs are in agreement with the respective field-observed hydrographs which confirms that the infiltration module is capable of capturing the infiltration behavior of soil.

5. Goodwin creek watershed simulation

The Goodwin creek watershed is an experimental watershed monitored by the national sedimentation laboratory, Oxford, Mississippi. This watershed is in the Yazoo river basin flowing in north Mississippi. The total drainage area of the watershed is 21.3 km^2 (King et al.1999). The outlet of the watershed is located at latitude 89^o 54' 50" and longitude 34^o 13' 55".

The watershed is monitored for investigating the land use and management practices that influence the rate and amount of nutrients transported to streams from the upstream areas. The watershed has 31 rain gauges located at various points. There are four classes of landuse, namely; cotton, forest, soybean and pasture. There are 7 classes of various combinations of soil types. The location of rain gauges, watershed boundary, landuse and soil type maps are shown in Figure 9. The influence of landuse is considered in the model as the resistance to flow in the form of spatial varying Manning's friction coefficient. The model reads the DEM files directly. A uniform computational grid of 30 m x 30 m is used. The time step is governed by the CFL condition. The boundary conditions are open type to all sides of the DEM.

The storm event dated October 17 and 18, 1981 is considered for calibration for this real life case. The starting time, storm duration and maximum rainfall intensity of the storm are different at different rain gauge locations (USDA-ARS-NSL Research report 1995). The storm started on October 17th and ended on October 18, 1981. Tables 3 and 4 show Manning's *n* for all land use and infiltration parameters for all soil types respectively. These values produced the closest match between the simulation and discharge measured at the watershed outlet. The comparison of simulated and observed runoff at the outlet of the watershed is shown in Figure 10. It can be observed that the storm hydrograph is reproduced by the model almost exactly. The peak discharge, duration and shape of the hydrograph are reproduced well.

Table 3

Sr. No.	Land use	Manning's *n*
1	Pasture	0.04
2	Forest	0.10
3	Soybean	0.03
4	Cotton	0.03

Table 4					
Sr. No.	Infiltration parameters / Soil type	Porosity	Effective Porosity	Dry Suction (cm)	Hydraulic Conductivity (cm/h)
1	Calloway	0.45	0.440	18.5	0.45
2	Collins	0.35	0.330	17.35	0.55
3	Fallaya	0.33	0.301	16.50	0.63
4	Grenada	0.35	0.310	16.80	0.49
5	Gullied land	0.55	0.486	16.68	0.65
6	Loring	0.52	0.486	16.68	0.65
7	Memphis	0.51	0.486	16.68	0.65

After calibration of the infiltration parameters using the storm on dated October 17 and 18, 1981, the model is applied to simulate a series of storms in continuation. The simulation period includes dry spells as well. Three storm events occurred consecutively on January 20, 21, 22 and 23 January, 1982. The discharge hydrograph, observed at the outlet, showed three peaks during these 4 days. The above calibrated parameters were used for simulation of these events. The comparison of the simulation and the field-observed flood hydrograph are shown in Figure 11. It can be observed that the model reproduced the field data for the first peak and the second peak was slightly under predicted. The third peak is also under predicted. However, the arrival time of the peaks in the hydrograph is captured well. This application shows that the model can be applied in a real life case for longer duration simulations as well.

5 Conclusions

A 2D shallow water model (Singh et al. 2007) employing the well-balanced, positivity-preserving Kurganov-Petrova (2007) scheme is extended to simulate overland flow by including rainfall and infiltration rates as source and sink terms. The Green-Ampt method is used for calculation of the infiltration rate. The model takes into account the spatial variability of rainfall and soil characteristics over the computational domain. Friction coefficients are assigned based on spatially varying land use. The model is calibrated by comparing the field experimental data and simulations. The parameters selected based on the calibration of the model, using part of experimental data, were used for the simulation of remaining field experimental storm events. A good match between the field observations and simulations is obtained.

The model is applied to simulate the real world case of Goodwin creek watershed. A heavy storm event in the watershed is used for calibration of the model parameters for the Goodwin creek watershed; then a series of consecutive storms are simulated using the calibrated model. The comparison of the runoff hydrograph resulting at the outlet of the watershed with the simulation shows a good match. This proves that the model is applicable to simulate the real life overland flow in a natural watershed resulting from heavy storms.

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